

Energy Barrier Topology and Disturbance-Sensitivity in Imperfect Open-Section Thin Shells

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This study investigates the influence of imperfections on the buckling response of an open-section thin-shell structure. A probing methodology is employed to quantify imperfection- and disturbance-sensitivity through the buckling energy barrier and characterize the underlying stability landscape. The results reveal that the natural buckle width, defined by the localized post-buckling deformation of the perfect shell, acts as a critical length scale governing disturbance-sensitivity. A transition in behavior is observed when the imperfection width becomes comparable to this intrinsic deformation scale. Imperfections wider than this length scale alter the post-buckling response and modify the buckling energy barrier, whereas narrower imperfections lead to a different response. This demonstrates that beyond modulating the buckling energy barrier, certain imperfections can also suppress the formation of post-buckling equilibria and increase the minimum buckling load. The results further show that the orientation of the imperfection (outward versus inward) plays an important role in determining the resulting stability landscape. Probing across the shell length reveals that outward imperfections shorten the disturbance-sensitive region, leading to more spatially deterministic buckle initiation, whereas inward imperfections broaden this region and produce a more spatially chaotic buckling initiation. These findings identify the natural buckle width as a governing parameter for disturbance-sensitivity and characterize how localized imperfections reshape the stability landscape of open-section thin-shell structures.

I. Nomenclature

R	=	Shell radius
t	=	Shell thickness
L	=	Shell length
θ	=	Shell Opening Angle

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α	=	Angular offset from the middle line
M_{cr-lin}	=	Linear buckling moment
$M_{buckling}$	=	Critical buckling moment
M_{min}	=	Minimum buckling moment
M_y^0	=	Preloading bending moment
F_p	=	Reaction force at the probe location
D_p	=	Probe displacement
Z_p	=	Probe location
Z_d	=	Dimple location
δ	=	Peak amplitude of the dimple
σ	=	Gaussian width of the dimple
EB	=	Energy barrier

II. Introduction

Ultra thin-shell composite booms are widely used in lightweight deployable structures due to their low mass and high packaging efficiency. Their extremely small thickness, however, makes their mechanical response highly sensitive to imperfections and disturbances [1–3]. Even minor deviations from the ideal geometry can considerably reduce their load-carrying capacity by lowering resistance to buckling. This sensitivity complicates the prediction of buckling behavior [4], as the energy barrier separating the pre-buckling and post-buckling states is often very small, allowing even weak perturbations to trigger premature instabilities [5, 6]. Understanding and controlling how imperfections influence stability is therefore central to improving the reliability and performance limits of thin-shell structures.

The unpredictable behavior and imperfection sensitivity of ultrathin shells have traditionally been addressed through empirical stochastic methods to establish lower bounds for buckling loads (knockdown factors [7, 8]). However, these methods present two key limitations. These methods often yield overly conservative load predictions, which reduces mass efficiency. In addition, they depend on destructive testing, limiting scalability and applicability to modern, complex material architectures [9].

In addition to the limitations of knockdown factor methods, predicting the post-buckling response introduces further challenges. At the onset of buckling, the shell undergoes a highly nonlinear transition involving complex interactions between global deformation, buckling eigenmodes, and geometric imperfections. This behavior, referred to as buckling localization [10, 11], allows multiple energetically comparable deformation paths to form, a condition often described as spatial chaos [12, 13]. Both the reduction of the energy barrier and the occurrence of localization remain extremely sensitive to imperfections, contributing to uncertainty in buckling prediction, particularly in structures with very low

thickness [2, 14].

Recent progress in structural stability research has shifted toward characterizing buckling resistance through energy barriers. A probing methodology was developed to map stability landscapes and identify the critical barriers that govern sensitivity to external disturbances, often referred to as shock-sensitivity [15–17]. Virost et al. [5] experimentally applied this approach to cylindrical shells, showing that the complex stability behavior of shells can be represented through a reduced low-dimensional description. Marthelot et al. [18] demonstrated similar capability in pressurized hemispherical shells, highlighting the method’s effectiveness in locating imperfection-sensitive regions. These results confirm that small-amplitude disturbances can easily overcome the energy barrier and trigger premature buckling [13]. Royer and Pellegrino [19] extended the probing methodology to ladder-type coilable booms, offering insight into the mechanics of more complex geometries. Subsequent studies broadened its application to cylindrical [20–22] and spherical [23] shells, as well as circular arches [24], cylindrical shell roofs [25], and pre-stressed stayed columns [26].

While early probing studies primarily examined disturbance sensitivity, later work demonstrated that localized geometric imperfections can significantly reduce the buckling energy barrier and lead to premature instability [6, 27–29]. The response of thin shells is strongly affected by both the magnitude and the spatial placement of defects. Ning and Pellegrino [30] investigated imperfection-insensitive configurations for axially loaded cylindrical shells. Derveni et al. [31] showed that, in pressurized hemispherical shells with multiple imperfections, the most severe defect dictates the critical buckling load. Cervi and Virgin [32] performed systematic analyses on cylinders with varying initial geometric imperfections, finding significant variations in buckling loads and deformation patterns. Lee et al. [33] further demonstrated that intentionally introduced imperfections can, under certain conditions, improve stability by redistributing stresses and increasing buckling resistance. These studies highlight that engineered imperfections can be intentionally applied to improve buckling strength and increase overall structural reliability.

In parallel to studies on engineered imperfection tailoring, extensive research has evaluated how imperfections reduce buckling strength. This behavior is commonly represented through knockdown factors [31, 34–39]. While these approaches effectively capture buckling load reduction, the specific influence of imperfection characteristics on the buckling energy barrier remains largely unexplored. Early work by Hutchinson and Thompson [6] analyzed spherical shells with localized dimples and showed that the energy barrier is highly dependent on imperfection amplitude. Royer and Pellegrino [19] later extended this concept to ladder-type coilable booms under bending, further illustrating the high sensitivity of the barrier to geometric imperfections.

The present work investigates how localized geometric imperfections reshape the stability landscape of open-section thin shells under bending. While previous studies have primarily examined imperfection effects through reductions in critical buckling load, i.e. knockdown factors, the present study maps the buckling energy barrier across the shell to reconstruct the topology of the stability landscape. A limited set of studies qualitatively explored the effect of imperfection amplitude on the buckling energy barrier at the critical buckling initiation site [6, 19, 40, 41]. This study

extends these initial observations to directly quantify the impact of imperfections on the shell's spatial sensitivity to disturbances and highlights the link between imperfection and disturbance-sensitivity. Furthermore, this work shows that the observed transition in energy barrier behavior aligns with the intrinsic natural buckle width, derived from the post-buckling deformation pattern, rather than with the linear buckling eigenmode wavelength. Building on our prior work [42–44], we perform a detailed numerical analysis incorporating both outward (also referred to as bumps [37]) and inward imperfections, and systematically assess their contrasting effects on the buckling energy barrier. The broader aim of this investigation is to establish a clearer understanding of how targeted geometric imperfections can be used to tailor stability characteristics and modulate the buckling response of thin-shell structures.

The structure of the paper is as follows. Section III presents the methodology, including the probing strategy for energy barrier quantification, the imperfection definition, and finite-element implementation. Section IV provides a comprehensive analysis of the results, focusing on the identification of the most instability-sensitive location, the influence of imperfection parameters on buckling resistance, and the resulting modification of disturbance-sensitive regions.

III. Methodology

This study focuses on the buckling response of an open cylindrical composite shell under bending. The geometry, boundary conditions, and loading configuration are shown in Figure 1. The shell has length L , thickness t , and radius R , with an opening angle θ . Geometric values are listed in Table 1. One end is fully constrained, while a constant bending moment M_y^0 is applied at the free end. The analysis procedure follows three main steps: (a) probing the buckling energy barrier, (b) locating the most sensitive buckling location, and (c) introducing a controlled geometric imperfection. A schematic representation of the procedure is illustrated in Figure 2, and a detailed explanation of each step is provided below.

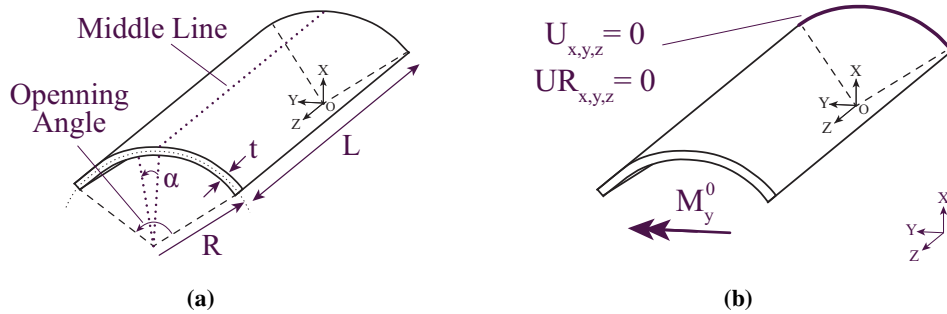


Fig. 1 Schematic of the (a) geometric parameters, (b) boundary conditions, and applied bending moment.

Table 1 Geometrical parameters of the open-section shell

Parameter	L (mm)	t (mm)	R (mm)	θ (deg)
Value	100	0.12	20	150

A. Probing and Quantifying the Buckling Energy Barrier

A probing methodology inspired by [5, 45] is employed to quantify the buckling sensitivity of the shell. The structure is first subjected to a constant bending moment M_y^0 , after which a controlled radial displacement D_p is applied at a selected location, and the corresponding reaction force F_p is recorded. Figure 2a presents this configuration, with Z_p indicating the axial probing coordinate. The resulting force–displacement response provides a direct measure of how sensitive the structure is to specific perturbations. In this curve (Figure 2d), a peak followed by a force drop to zero marks the transition from stable to unstable equilibria. This signifies the onset of instability, which corresponds to the formation of a localized dimple [18]. The buckling energy barrier is computed as the area enclosed by the hump in the $F_p - D_p$ curve. This represents the mechanical work required for the shell to leave the pre-buckled state and transition into a buckled configuration [16]. In addition to quantifying the energy barrier, this approach detects the lowest load at which buckling can be triggered by perturbations. This minimum load is lower than the critical value predicted by a linear bifurcation analysis [5].

B. Finding the Most Sensitive Buckling Location

To identify where imperfections most strongly influence stability, the buckling energy barrier is evaluated as a function of the probing location along both the axial and transverse directions (Z and α in Figure 1a) of the shell. Figure 2b schematically illustrates the location of the minimum energy barrier, represented by Z_{cr} . This minimum energy indicates the most instability-sensitive location, where a geometric imperfection would have the greatest influence on buckling.

C. Imperfection Definition

Following identification of the most sensitive location, a localized dimple is introduced directly at that point to evaluate how a prescribed imperfection influences the minimum buckling energy barrier and the resulting buckling response (Figure 2c).

The imperfection is modeled as a single dimple. This type of imperfection provides the lowest-energy pathway from the pre-buckled state to a buckled configuration before the bifurcation, forming mountain pass points in the shell’s energy landscape [4]. In practice, these mountain passes are the first to be eroded by imperfections in a real structure and are therefore critical. Local buckles taking the shape of a single dimple have also been observed in other close-section [46, 47] and open-section [19] geometries. We hypothesize that this also represents a physically relevant imperfection shape for the present geometry. While shallow open-section shells are typically governed by snap-through

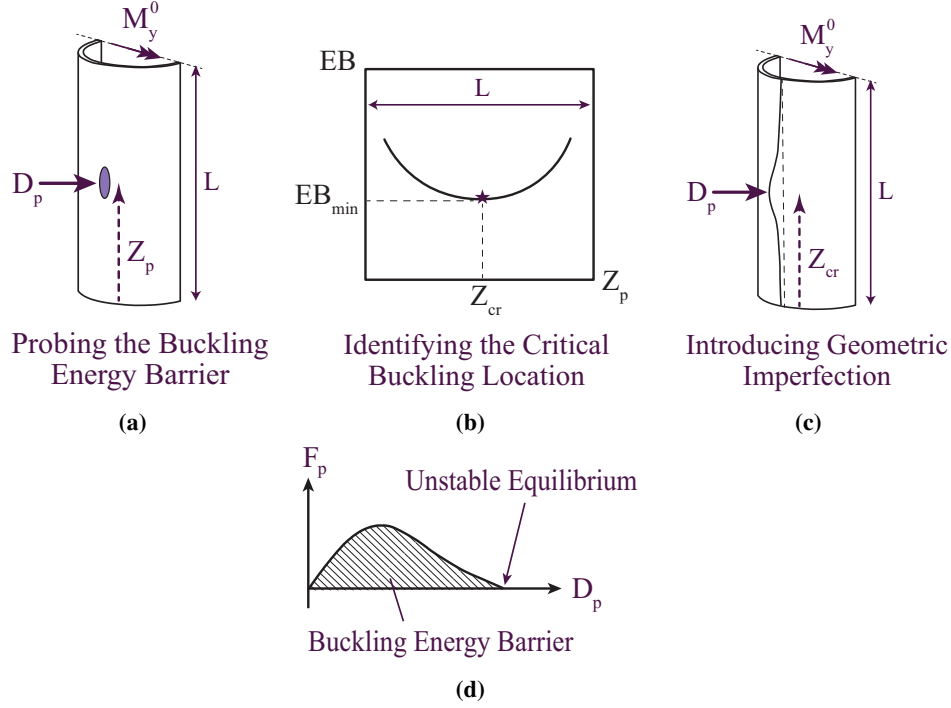


Fig. 2 (a-c) Schematic representation of the methodology, (d) definition of the buckling energy barrier for a fixed bending moment.

instabilities, the deeper shells considered here exhibit a localized buckling response that more closely resembles that of cylindrical shells. Because buckle initiation is governed primarily by the local geometry and deformation in the vicinity of the buckle, we expect the buckling response to depend more strongly on the local curvature rather than the transverse free-edge conditions. Accordingly, a single localized dimple is adopted in the present study as a representative imperfection shape for evaluating the influence of localized imperfections on shell stability. However, it is acknowledged that the criticality of this imperfection for this specific geometry has not been rigorously proven and is beyond the scope of this paper.

The imperfect structure was obtained by conducting a static analysis in which the displacement of the structure's centerline, following the shell normal, takes the shape of a Gaussian function,

$$\omega = \delta e^{-\frac{(Z-Z_d)^2}{2\sigma^2}} \quad (1)$$

where δ denotes the peak amplitude, with positive values ($\delta > 0$) producing an outward (positive) dimple and negative values ($\delta < 0$) an inward (negative) dimple. Z_d specifies the axial coordinate of the peak, and σ represents the Gaussian standard deviation, which is proportional to the longitudinal dimple width (Figure 3). In the circumferential direction, the normal imperfection displacement decays until coinciding with the nominal circular cross-section, and is purely dictated by the shell equilibrium configuration when subjected to the centerline boundary condition. As a

result, the circumferential width and the resulting circumferential-to-axial aspect ratio are not explicitly prescribed in this analysis, which is a limitation of the current study. Future work will fully prescribe 2D displacement fields to introduce the imperfection. The modified geometry is then imported into the finite-element model and subjected to the same probing procedure as the perfect structure. The resulting energy barrier distribution quantifies how imperfection characteristics influence the buckling behavior.

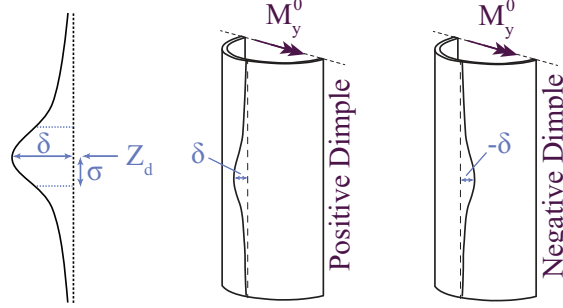


Fig. 3 Illustration of the Gaussian field used to define the imperfection geometry, and the associated dimple configurations.

D. FEM Implementation

All numerical simulations are implemented in Abaqus/standard using reduced-integration S4R shell elements within a finite-strain formulation. The final mesh contains 5,200 elements, determined through a mesh-convergence study performed in our earlier work [43]. The structural layup consists of a symmetric [0/90/0] carbon fiber reinforced laminate (CFRP, MR60H/PMT-F7). Each ply is represented as an orthotropic elastic lamina and homogenized using Classical Lamination Theory. The associated material properties [48] are listed in Table 2. The analysis is performed in two nonlinear static steps. First, a bending moment M_y^0 is applied using Newton-Raphson solver (static general method in Abaqus) to preload the structure. The bending moment is applied to a reference point rigidly coupled to the free edge of the shell (Figure 1a). In the second step, this moment is held constant while a radial displacement is imposed at a single node, and the corresponding reaction force is recorded. The displacement is incrementally increased until instability is detected. Newton–Raphson was initially employed for this step; however, instabilities were encountered during probing, leading to a vertical tangent and a loss of convergence [43]. To overcome this limitation, an arclength solver (modified Riks method in Abaqus) is employed instead, enabling the analysis to proceed beyond the onset of instability.

Table 2 Material properties of the unidirectional lamina used in the composite shell model [48].

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	t (mm)
CFRP	174.40	8.39	6.40	0.259	0.04

IV. Results and Discussion

A. Determining the Most Buckling-Sensitive Location

The variation of the buckling energy barrier is assessed by sequentially probing the shell at locations along the axial and transverse directions. The main objective is to identify the location with the lowest energy barrier, which corresponds to the most buckling-sensitive location. The minimum energy barrier is observed at the symmetric mid-span of the shell ($\alpha = 0^\circ$ and $Z = 50 \text{ mm}$), indicating that this is the most probable location for buckling initiation. However, analysis of the post-instability response further reveals that, due to buckle propagation, the buckle initiation location does not always correspond to the buckle equilibrium location. Regardless of the initial probing location, the buckle progressively migrates toward the mid-span and ultimately stabilizes at $Z = 50 \text{ mm}$ in the stable equilibrium state. These observations confirm this coordinate as the critical buckling location. Introducing an imperfection at this location is therefore expected to have the most significant impact on the shell's global buckling behavior. A more detailed discussion on the identifying this critical location is provided in our previous works [42, 43].

B. Critical Imperfection Parameters Governing Buckling

To understand how geometric imperfections influence buckling stability, a single Gaussian dimple is introduced at the critical buckling location, $Z_d = 50 \text{ mm}$. The energy barrier is then measured at the same point $Z_p = 50 \text{ mm}$ over a range of imperfection widths σ/L and amplitudes δ/t . Probing directly at the dimple location ($Z_d = Z_p$) ensures that the measured response reflects the local influence of the imperfection. The resulting response, shown in Figures 4a and 4b, illustrates how variations in dimple width and amplitude modify the buckling energy barrier under a constant bending moment of $M_y^0 = 1.4 \text{ Nm}$, corresponding to a load level close to the minimum buckling load.

Focusing on the impact of imperfection height, outward and inward dimples exhibit opposite effects on structural stability. Increasing the amplitude of a positive imperfection has a stabilizing influence: the buckling energy barrier rises with δ/t , indicating that larger outward dimples require greater perturbation to trigger buckling. In contrast, negative dimples display a destabilizing response, in which increasing δ/t decreases the energy barrier and therefore elevates the likelihood of premature buckling.

Turning to the effect of imperfection width, the response for positive imperfections (Figure 4a) exhibits three distinct regimes. In the first regime ($\sigma/L < 0.05$), increasing the width leads to a gradual rise in the energy barrier. In the second regime, the curves flatten into a plateau where the energy barrier becomes independent of σ/L . Finally, beyond a certain threshold ($\sigma/L = 0.07$), no post-buckling equilibrium can be identified, meaning that the energy barrier can no longer be defined. In this regime, the imperfection is wide enough to suppress buckle formation, effectively increasing the minimum load required for instability. A more detailed discussion of the physical mechanisms underlying these three regimes is presented in our previous work [43].

The response of negative dimples (Figure 4b) also exhibits the same width threshold $\sigma/L = 0.07$, but in a mirrored

way relative to the positive case. When the imperfection width is below this value, no post-buckling equilibrium is observed at the applied load ($M_y^0 = 1.4 \text{ Nm}$), as the probe force–displacement response does not cross the zero-force threshold. This indicates that the imperfection suppresses the initiation of post-buckling equilibria at the specific probing location, and for the specific moment $M_y^0 = 1.4 \text{ Nm}$. By definition, this suggests an increase in the minimum buckling load, which is discussed further in Section IV.E). This behavior differs from the buckling load response of negative dimples. While negative dimples are generally associated with reductions in buckling load, the present results reveal width-dependent trend for the minimum buckling load. Above the critical width, the energy barrier remains relatively insensitive to σ/L , demonstrating a plateau behavior consistent with that observed for positive dimples.

To further interpret the observed energy barrier trends in Figures 4a and 4b, each data point is classified according to its probing response. The marker assigned to each point represents a distinct buckle formation mechanism. The corresponding probe force-displacement curves and deformed configurations are presented in Figures 4c-4f, where four distinct buckling types (Types I-IV) are identified and described below.

Type I ($\square$): Stationary buckle formation. Type I corresponds to buckling through a stable probing response under displacement control. As shown in Figure 4c, the shell initially resists lateral probing with positive stiffness until the probe force reaches its local maximum value (termed ridge). The deformed configuration at this point reveals a localized buckle centered at the probe location, $Z = 50 \text{ mm}$. As the probe displacement increases beyond the ridge and the response enters the negative stiffness regime, the localized buckle increases in amplitude while remaining centered at the same location ($Z = 50 \text{ mm}$). The response then reaches the unstable equilibrium point without buckle propagation or snap through. This response therefore provides a well-behaved path to the unstable equilibrium configuration, with the buckle forming and remaining at the same location throughout the probing process. This buckling type ($\square$) is the predominant buckle formation mode for the inward imperfections shown in Figure 4b over the parameter space considered.

Type II ($\diamond$): Type II initially follows the same response as Type I. Beyond the ridge, as the response enters the negative stiffness regime, the localized buckle continues to grow while remaining centered at $Z = 50 \text{ mm}$. However, before the probe force reaches zero, the equilibrium path undergoes snap-back associated with buckle propagation (Figure 4d). During this transition, the localized buckle relocates from its initial site ($Z = 50 \text{ mm}$) to a neighboring energetically preferred location. As demonstrated in our previous work [43], the buckle propagates toward the location of minimum energy barrier location, which acts as an attractor in the stability landscape. This behavior was experimentally observed for cylindrical shells [49] and reveals the existence of basins of attraction around energy barrier minima. This behavior agrees with the spatial energy barrier topology (discussed in Section IV.C), for which the single energy barrier minimum for a perfect structure splits into two minima, therefore two basins of attractions, in the presence of an outward imperfection. Following the buckle propagation, the response continues toward the unstable equilibrium point, which is reached at zero probe force with the buckle centered at $Z = 55 \text{ mm}$. Unlike Type I, where the unstable equilibrium

is attained through continuous growth of a buckle at a fixed location, Type II involves buckle propagation through snap-back [5, 49], causing the buckle to relocate before the final unstable equilibrium configuration is reached. Type II is observed only at relatively small imperfection amplitudes for outward imperfections (Figure 4a) and at relatively small imperfection widths for inward imperfections (Figure 4b).

Type III ($\circ$): Type III follows the same response path as Type II until buckle propagation occurs. The key distinction is that, unlike Type II, the equilibrium path restabilizes (under displacement control) before reaching the unstable equilibrium (Figure 4e). This buckling formation type ($\circ$) is the dominant buckling response over most combinations of imperfection width and amplitude for positive dimples (Figure 4a).

Type IV (Δ): Type IV follows a response similar to that of Type III, except that buckle propagation is initiated at the local probe force maximum (ridge). A further distinguishing feature of Type IV is that, as shown in Figure 4f, the buckle is no longer fully localized at the ridge. Instead, two competing localization sites emerge: the primary site at $Z = 50 \text{ mm}$, corresponding to the probe location, and an energetically favorable site located at ($Z = 42 \text{ mm}$). Type IV (Δ) is observed only for the outward imperfections with relatively large amplitudes combined with small widths (Figure 4a). The energy competition between the two localization sites drives the buckle to propagate to the minimum energy barrier location. The response then continues toward the unstable equilibrium point, where the fully developed buckle is centered at ($Z = 55 \text{ mm}$). In contrast, for inward imperfections (Figure 4b), type IV (Δ) is not observed because inward imperfection further decreases the minimum energy barrier at the center of the structure, which remains the center of a single basin of attraction.

The buckling classification presented above reveals that the non-monotonic behavior in Figure 4a, including the curve crossing at $\sigma/L = 0.04$ for amplitudes of $\delta/t = 0.92$ and $\delta/t = 1.08$, corresponds physically to a transition in the underlying buckle formation mechanism captured by the solver. To further characterize this transition, the probe force-displacement response for the two intersecting cases are computed up to the onset of buckle propagation, after which a linear perturbation (eigenvalue) analysis is performed (Figure 5). In both cases, an eigenmode with a near-zero eigenvalue is identified immediately before the propagation point, and therefore multiple equilibrium paths exist in the vicinity of the propagation point. For the $\delta/t = 0.92$ case (Figure 5a), the Newton-Raphson solver follows an alternative equilibrium branch after the bifurcation and converges to the unstable equilibrium through the Type I response, whereas an arc-length solver captures the Type III propagation path. This demonstrates that both paths coexist for the same structure and loading conditions. In contrast, for the $\delta/t = 1.08$ case (Figure 5b), the Newton-Raphson solver encounters instability immediately after bifurcation and is unable to converge to an alternative equilibrium branch. Consequently, only the Type IV pathway is obtained numerically, although the bifurcation indicates that additional equilibrium pathways are expected to exist. To capture all the possible probing responses, a more complicated path-following algorithm may be employed [50], which is beyond the scope of the present work.

To investigate the origin of the critical width, the natural buckle shape of the perfect shell, defined as the intrinsic

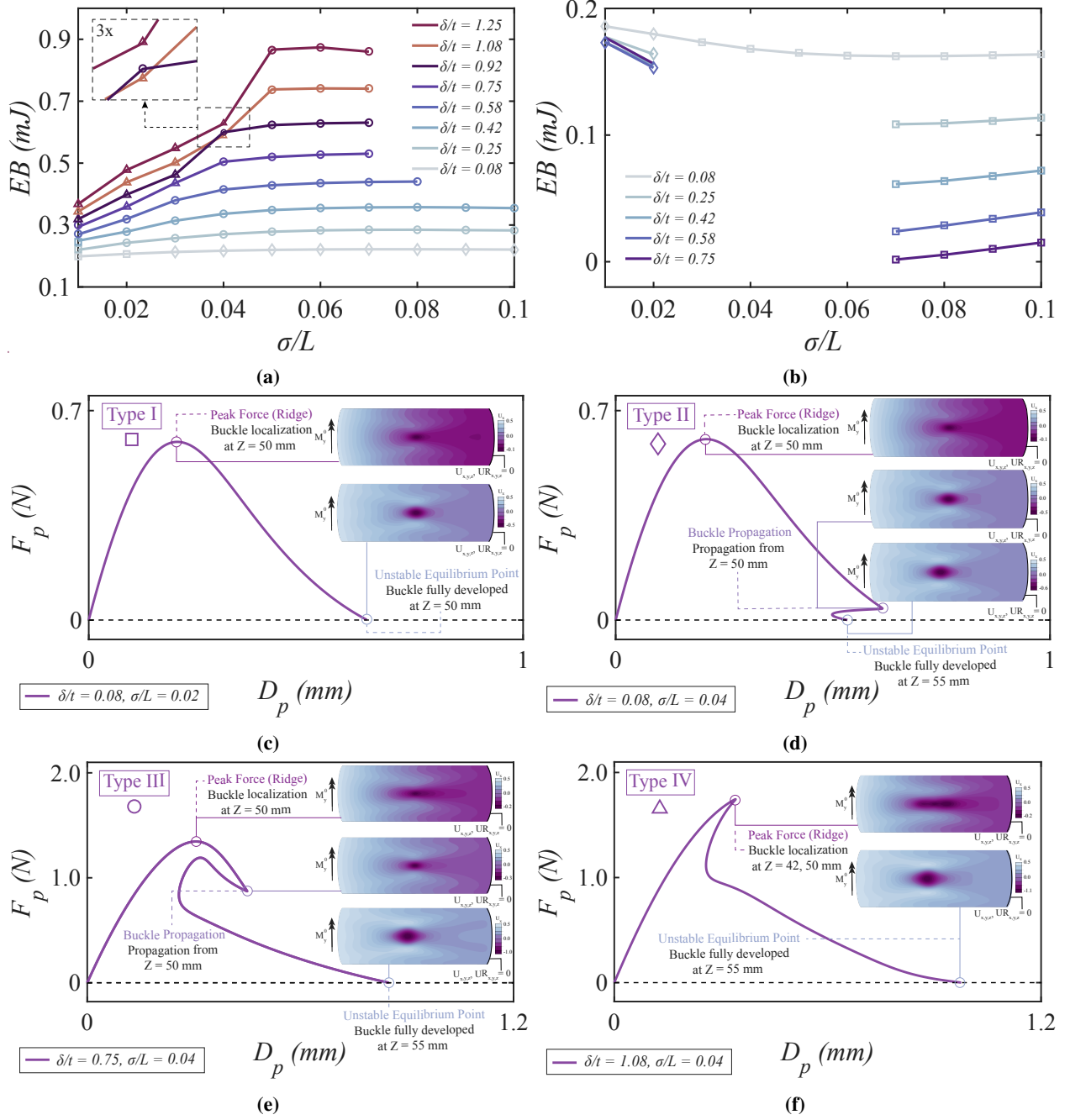


Fig. 4 Energy barrier variation with imperfection width: (a) outward, (b) inward imperfection; (c–f) representative probe force–displacement responses for the four buckling types (outward imperfections).

width of the localized post-buckling deformation, is extracted under the same applied moment (Figure 6). The natural buckle characteristics are obtained using a disturbance-based approach to transition the structure into the post-buckling regime. The shell is probed until a stable post-buckling equilibrium is reached, after which the probe is released, and the reaction force returns to zero. At this stage, the structure remains in its stable equilibrium state without any external disturbances. The resulting deformed profile is then extracted and fitted with a Gaussian function curve. The deformed

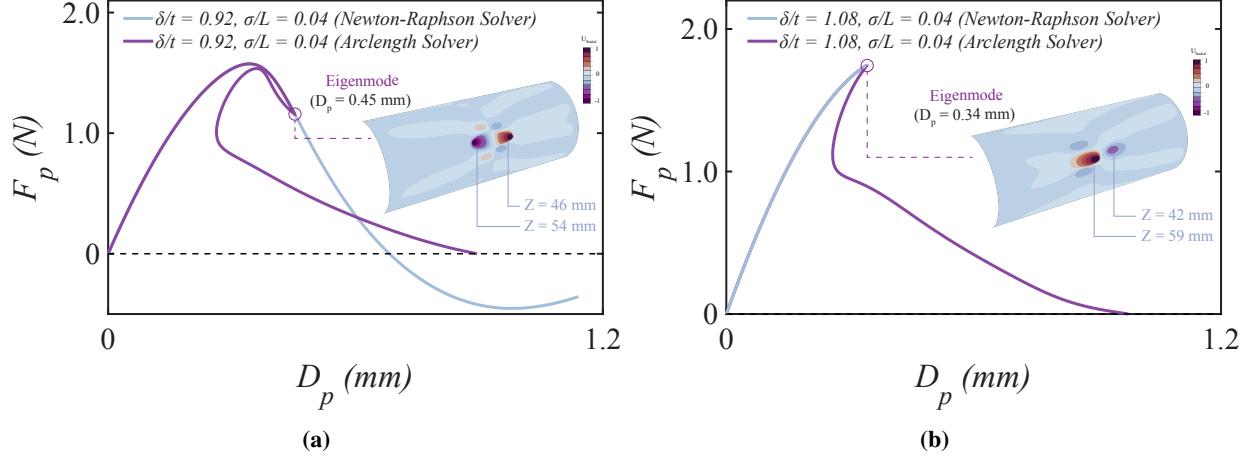


Fig. 5 Bifurcation near the propagation point for (a) Type III and (b) Type IV, illustrating multiple equilibrium paths toward the unstable equilibrium configuration.

configuration at the stable equilibrium state, along with a lateral view of the longitudinal midsection, is shown in Figure 6a. The fitted curve shown in Figure 6b yields a characteristic width of $\sigma/L = 0.07$, which matches the critical value observed in Figure 4. The corresponding Gaussian width is defined here as the natural buckle width. This indicates that the transition in stability response occurs when the imperfection width becomes comparable to that of the natural buckle. This observation is derived from analysis at a single applied moment close to the minimum buckling load, and further investigation is required to evaluate its validity across different loading conditions.

Prior studies have shown that local dimple imperfections have the strongest effect on the buckling load when their width becomes comparable to the characteristic wavelength of the buckling mode [51]. However, the influence of imperfection width on the buckling energy barrier and minimum buckling load, which are investigated in the present work, remains largely unexplored in previous studies. In addition to the natural buckle characteristics (Figure 6), the characteristic eigenmode wavelength is also investigated to contrast the present results with prior literature. The deformed shape obtained from a linear buckling analysis, featuring a global distribution of inward and outward waves, is illustrated in Figure 7a. To obtain the width of the buckling mode, the deformation profile of the half-wave associated with the maximum inward radial displacement is first extracted. The characteristic length scale of the buckling mode is then approximated using a Gaussian function, as shown in Figure 7b.

Comparison between the critical width threshold ($\sigma/L = 0.070$), the natural buckle width ($\sigma/L = 0.070$), and the characteristic eigenmode Gaussian width ($\sigma/L = 0.037$) shows that the critical energy barrier width threshold aligns more closely with the intrinsic width of the post-buckling deformation rather than the linear eigenmode wavelength. This difference is likely related to the loading state considered here. Both the critical width threshold and the natural buckle width are obtained under an applied moment of $M_y^0 = 1.4 \text{ Nm}$, which is close to the minimum buckling load and therefore well below the bifurcation point. Near the bifurcation, the shell deformation is still well represented by the

linear eigenmode (see Section IV.D). However, nonlinear deformation and localization effects cause the buckle shape to dramatically evolve away from the bifurcation [28]. As a result, the post-buckling deformation scale, represented here by the natural buckle width, seems to dominate the buckling energy barrier response close to the minimum buckling load.

Figure 4 further illustrates the influence of imperfection amplitude on the critical width threshold. For positive dimples, excluding the lowest-amplitude cases ($\delta/t < 0.5$), the critical width at which the energy barrier vanishes remains approximately constant across the investigated amplitudes. A similar behavior is observed for negative dimples. Except for the very low amplitude case of $\delta/t = 0.08$, which has only a minor influence on the buckling energy barrier, the width threshold also remains approximately independent of dimple amplitude. The results therefore suggest that the threshold is primarily governed by the natural buckle width for the loading at which the energy barrier is computed, rather than by the imperfection amplitude itself. Since the natural buckle width depends on the applied loading (here M_y), variations in loading magnitudes are expected to influence the critical energy barrier threshold.

Overall results demonstrate that when both σ/L and δ/t are very small, they do not meaningfully alter the energy barrier compared with the perfect shell, and thus have negligible influence on the buckling response. Excluding these minor cases, the results indicate that positive imperfections postpone buckling when their width exceeds the natural buckle width, whereas negative imperfections exhibit the same stabilizing effect when their width is below this threshold.

It is worth noting that while the quantitative values reported here, including the critical imperfection width and buckling energy barrier magnitude, depend on the investigated shell geometry, boundary conditions, and loading configuration, previous studies on spherical shells [6] and ladder-type structures [19] have shown similar qualitative trends in energy-barrier evolution across different geometries. This suggests that the broader physical mechanisms identified here may extend to other imperfection-sensitive shell structures, although further work is needed to assess the generality of these observations across different geometries, boundary conditions, and loading configurations.

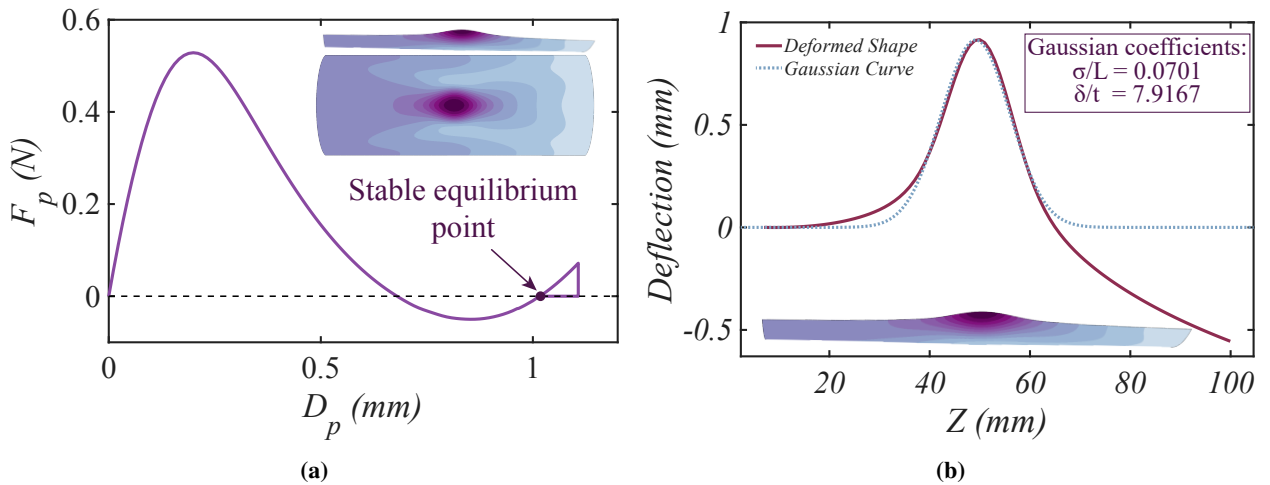


Fig. 6 Natural buckle characteristics: (a) probe response and corresponding deformed shape at stable equilibrium, (b) fitted gaussian curve to the deformed shape profile.

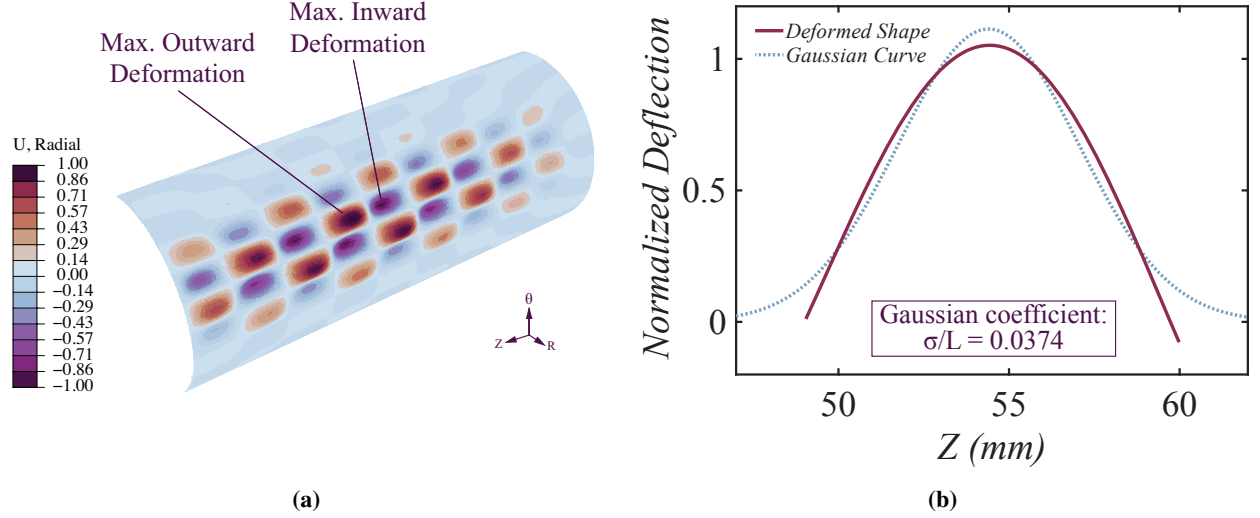


Fig. 7 Characterization of the eigenmode length scale: (a) buckling eigenmode, (b) Gaussian fit of the dominant half-wave.

C. Imperfection Influence on the Disturbance-sensitive Region

In the previous section (IV.B), the local effect of imperfections was examined by evaluating the energy barrier only at the imperfection location ($Z_d = Z_p$). In this section, the focus shifts to investigating how a localized imperfection affects buckling sensitivity along the entire shell length. To achieve this, a single dimple is introduced at the critical buckling location ($Z_d = 50 \text{ mm}$), while the probing location is varied across the full span of the structure ($0 \leq Z_p \leq 100 \text{ mm}$). This approach enables quantification of how far the imperfection effect extends, providing a more global measure of disturbance sensitivity. The corresponding results are presented in Figure 8. Note that the imperfection width is fixed at ($\sigma/L = 0.07$), corresponding to the critical width identified in the previous section and equal to the natural buckle width of the perfect shell. Although the same analysis can be performed for other imperfection widths, this value is selected because it corresponds to the transition between the two distinct responses identified in Figure 4.

Each curve in Figure 8 illustrates the buckling energy barrier as a function of probing location along the shell, while the imperfection location remains fixed at the mid-span. In Figure 8a and 8b, the buckling energy barrier is defined only over a limited range of Z_p , and this range represents locations where a post-buckling equilibrium exists. Outside this interval, no buckled equilibria are obtained, meaning the structure remains stable and the energy barrier cannot be defined at those coordinates for the specific applied moment. Accordingly, the length of each curve represents the portion of the shell's length that is sensitive to disturbance-induced buckling, which is referred to here as the disturbance-sensitive region (also called shock-sensitive region [15–17]).

Focusing on the perfect structure response, buckling can be initiated over a relatively wide span of the shell (see Figure 8c for the quantitative comparison), indicating that a large portion of the structure is disturbance-sensitive. It is worth noting that while buckling is initiated within this range, the equilibrium location may differ from the initiation

location yet still remain within this same interval.

When a positive dimple is introduced (Figure 8a), the disturbance-sensitive region contracts noticeably. This indicates that the minimum buckling load required to trigger instability increases at locations outside the sensitive region compared to the perfect structure. For small amplitudes, buckling can still be initiated over a substantial portion of the structure, although the range is already reduced relative to the perfect shell. At higher amplitudes, the disturbance-sensitive interval becomes highly localized around the imperfection location (see Figure 8c for the quantitative comparison). Positive imperfections therefore act as a mechanism for localizing buckling. This behavior demonstrates that positive dimples decrease global sensitivity by elevating the disturbance energy required to induce buckling. Thus, the system transitions from a spatially chaotic [13] and broadly sensitive configuration to one in which buckle formation becomes spatially deterministic and constrained.

Examining the effect of a negative dimple (Figure 8b) on the disturbance-sensitive region reveals a response opposite to the positive case. Rather than narrowing, the disturbance-sensitive length expands beyond that of the perfect shell, resulting in a broader range of locations at which buckling can be initiated. This behavior indicates a more chaotic and spatially less predictable response. For positive dimples, the energy barrier distribution is characterized by a distinct peak with a high energy barrier at the imperfection point. In contrast, negative dimples do not create a local minimum of energy barrier. Instead, they generate a plateau across the central portion of the curve. Within this plateau, the buckling energy barrier remains uniformly low and independent of the probe location. This behavior indicates that negative dimples expand the disturbance-sensitive region while simultaneously reducing the local resistance to buckling. As a result, even minor perturbations can trigger instability across a broad portion of the structure, amplifying uncertainty about where buckling will ultimately initiate and making the response more unpredictable. The factors controlling the length and extent of the plateau are not yet fully understood, and further study is required to identify which geometric or loading parameters influence the size of this region.

D. Sensitivity of the Critical Buckling Load to Imperfection Geometry

The influence of geometric imperfections on buckling strength has been extensively studied, with their effects commonly expressed through buckling knockdown factors [34–39]. These approaches effectively describe the reduction in load-carrying capacity associated with imperfections. Accordingly, the critical buckling load is widely adopted as the principal criterion for stability assessment in thin-shell structures. This section examines the quantitative relationship between the critical buckling load and imperfection size. To this end, a localized negative dimple is introduced at the critical buckling location with varying amplitudes. An increasing bending moment is then applied using an arc-length (Riks) solver to trace the post-buckling equilibrium path and determine the corresponding critical buckling load, as illustrated in Figure 9a. The moment-rotation post-buckling responses of imperfect shells with a fixed normalized imperfection width of $\sigma/L = 0.08$ and varying amplitudes are presented in Figure 10a. The results show a strong

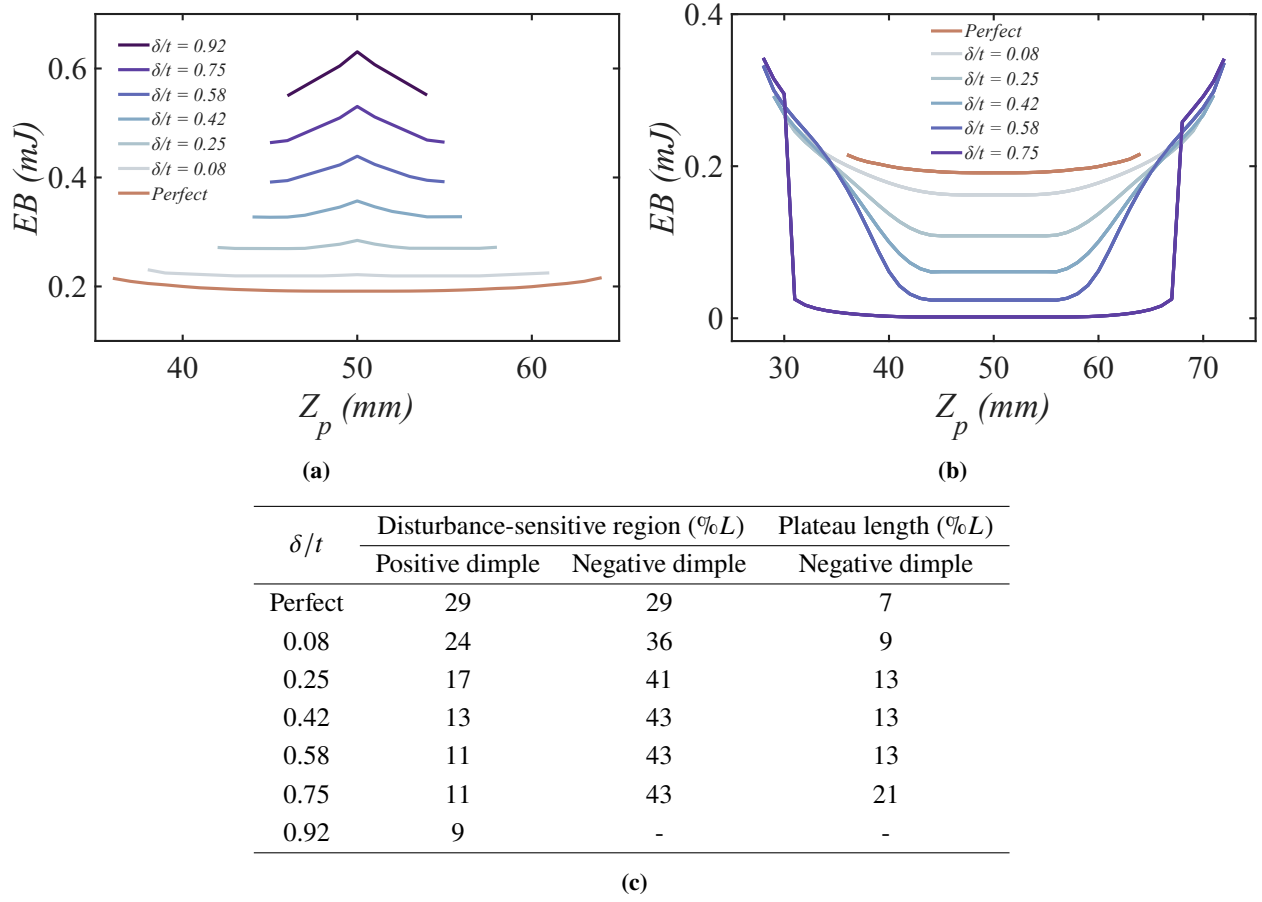


Fig. 8 Energy barrier along the length under $M_y^0 = 1.4 \text{ Nm}$ for $\sigma/L = 0.07$, (a) positive dimple (taken from [43] copyright © 2025 AIAA) (b) negative dimple (c) quantitative comparison.

sensitivity of the buckling load to imperfection amplitude, with larger amplitudes producing a noticeable reduction in buckling strength for a given dimple width. The influence of imperfection amplitude on the critical buckling load follows the same trend reported in previous studies [6, 33]. Figure 10b presents the corresponding critical buckling loads as a function of the normalized imperfection width. For each amplitude level, the buckling load initially decreases as the dimple width increases, reaches a minimum at a specific width, and subsequently increases for larger widths. This behavior demonstrates a non-monotonic sensitivity of the critical buckling load to imperfection width. Among the investigated configurations, the imperfection characterized by the normalized amplitude of $\delta/t = 0.75$ and the normalized width of $\sigma/L = 0.06$ yields the most severe reduction in buckling strength, decreasing the critical buckling load by 50% relative to the perfect structure ($M_{buckling} = 2.78 \text{ Nm}$).

Existing literature identified the linear buckling mode wavelength as the critical length scale at which the effect of an imperfection is maximized. Our results for the buckling load (Figure 10b) align with this common expectation. For lower imperfection amplitudes ($\delta/t < 0.75$) the lowest value of buckling moment is consistently found for σ/L between 0.03 and 0.04. Figure 7 highlights that the half-wavelength of the eigenmode is approximately $\sigma/L = 0.037$, which

aligns with the buckling load minima of Figure 10b.

The influence of imperfection amplitude on the buckling load is comparatively systematic, with larger amplitudes leading to progressively greater reductions in buckling capacity. In contrast, the effect of imperfection width is non-monotonic, the buckling load decreases with increasing dimple width up to a characteristic width at which the minimum load is attained, after which the buckling load increases for larger widths.

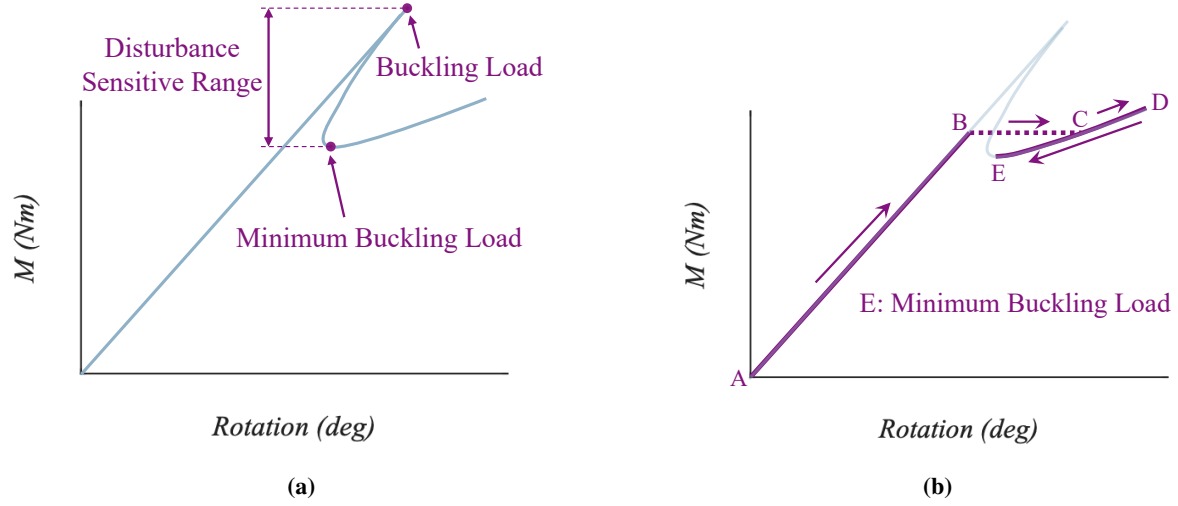


Fig. 9 Schematic illustration of (a) the conventional monotonic loading procedure and (b) the probing procedure used to determine the minimum buckling loads.

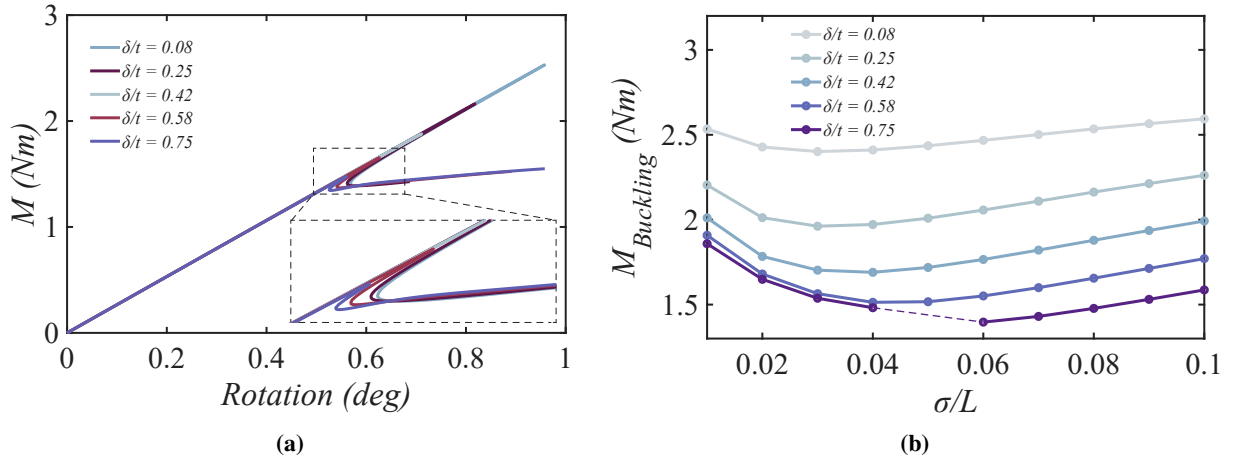


Fig. 10 Sensitivity of buckling strength to inward imperfections: (a) moment–rotation response for fixed $\sigma/L = 0.08$ and varying amplitudes, (b) variation of the critical buckling load.

E. Sensitivity of the Minimum Buckling Load to Imperfection Geometry

Recent studies have increasingly adopted the minimum buckling load (Figure 11a) as an alternative buckling criterion for characterizing the stability of thin-shell structures. In particular, prior work has demonstrated that this quantity

provides a tight lower bound to experimental buckling loads, allowing the estimation of buckling knockdown factors without large-scale destructive testing campaigns [52].

In this section, the sensitivity of the minimum buckling load to the size of inward imperfections is investigated. Although the effect of imperfections on the classical critical buckling load has been extensively studied and is generally found to decrease monotonically with increasing imperfection amplitude, the evolution of the minimum buckling load is shown here to be significantly more complex and strongly dependent on the detailed characteristics of the introduced imperfections.

To determine the minimum buckling load, instead of the conventional procedure of monotonically increasing the applied moment until transition to the post-buckling regime (Figure 9a), a localized disturbance is applied at a load level below the critical buckling load to trigger an early transition of the structure into the post-buckling regime (Figure 9b). In this procedure, the structure is first loaded by applying an increasing bending moment up to point B , corresponding to a pre-buckling equilibrium state below the critical load. A localized disturbance is then applied (path $B - C$), driving the structure toward the post-buckled equilibrium at point C . From this state, the applied moment is subsequently increased to point D and then gradually reduced until the minimum buckling load is reached at point E . This procedure is applied to both perfect and imperfect structures, and the corresponding results are presented in Figure 11. The moment–rotation responses for the perfect and selected imperfect shells are shown in Figure 11a. Analysis of the post-buckling regime indicates that, despite differences in imperfection characteristics, all the imperfections lead to post-buckling solutions that converge to the same stiffness and large-deflection response. Figure 11b illustrates the dependence of the minimum buckling load, M_{min} , as a function of the normalized imperfection width on the normalized imperfection width σ/L for different imperfection amplitudes δ/t .

Two distinct regimes are observed. In the first regime, corresponding to small imperfection widths, increasing the imperfection amplitude leads to an increase in the minimum buckling load, indicating a stabilizing effect, i.e. leading to less disturbance-sensitive structures. In the second regime, associated with larger imperfection widths, increasing the amplitude results in a decrease in the minimum buckling load, reflecting a destabilizing influence, i.e. a largest range of moments for which the shell is metastable. The transition between these two behaviors occurs at a critical value of σ/L , which is to first order independent of the imperfection amplitude. Strikingly, this critical value is close to the natural buckle width, $\sigma/L = 0.07$, found in the post-buckling regime of the perfect structure at $M = 1.39 \text{ Nm}$ (minimum buckling load for that structure). In the corresponding plot, a data point is absent for $\delta/t = 0.75$ and $\sigma/L = 0.05$, corresponding to a condition in which the actual buckling load falls below the minimum buckling load, and the unstable bifurcation no longer exists, as illustrated in Figure 11a.

Overall, the critical buckling load exhibits significantly greater sensitivity to geometric imperfections than the minimum buckling load. Among the configurations investigated, the imperfection with normalized amplitude of $\delta/t = 0.75$ produces the largest variation in the minimum buckling load, leading to a maximum increase of

approximately 6% at $\sigma/L = 0.04$ and a maximum decrease of about 4% at $\sigma/L = 0.10$, relative to the perfect structure ($M_{min} = 1.39 \text{ Nm}$). These variations are substantially smaller than those observed for the critical buckling load, where comparable imperfections can lead to reductions of up to 50%, highlighting the comparatively lower imperfection sensitivity of the minimum buckling load.

The surprising evolution of the minimum buckling load as a function of imperfection parameters may be specific to this geometry/loading and the generality of this observation is not implied here. Future work will specifically investigate how the local strain energy density varies as the shell deforms from its initial imperfect shape to an inward buckle and investigates whether the non-linear pre-buckling deformation of the imperfection (growth or decay) may provide a physical mechanism that explains this minimum buckling load trend.

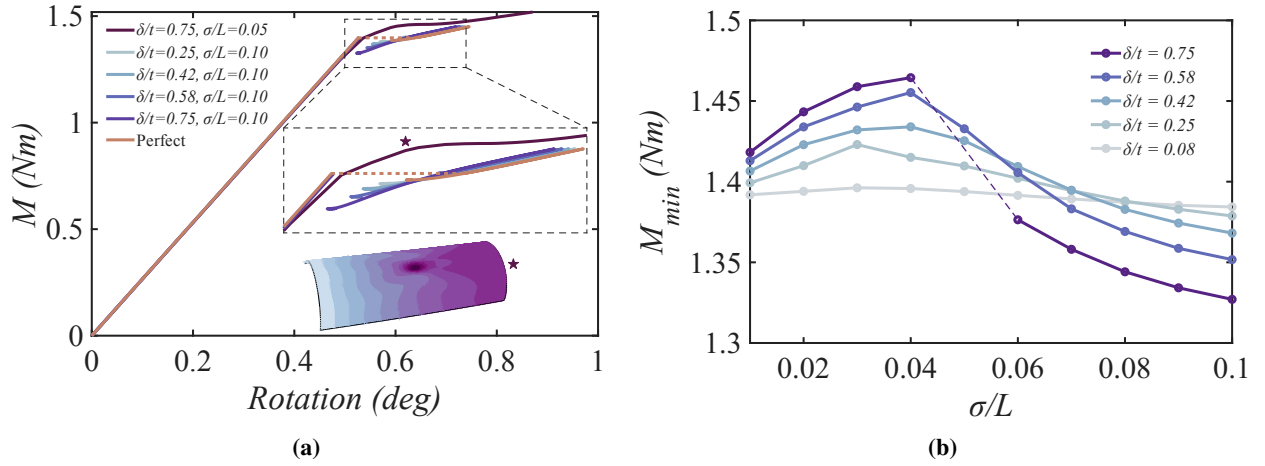


Fig. 11 Effect of inward imperfections on the (a) post-buckling response (b) minimum buckling load.

F. Disturbance-Sensitive Load Range

The disturbance-sensitive load range is defined as the moment interval between the minimum buckling load and the critical buckling load, as schematically illustrated in Figure 9a. Within this interval, the structure remains stable under monotonic loading; however, the pre-buckling equilibrium is metastable, meaning that the structure can sustain the applied load in the absence of disturbances but may undergo a transition to the post-buckled configuration when subjected to sufficiently large perturbations. As discussed by [16], the unbuckled state at subcritical loads remains stable under small perturbations but can lose stability when subjected to disturbances of sufficient magnitude. In this range, the probability of premature buckling depends on the magnitude of the potential energy that must be overcome to transition from the unbuckled configuration to the unstable post-buckled state. Consequently, the disturbance-sensitive load range provides a measure of the sensitivity of the structure to buckling under disturbances and quantifies the difference between the classical critical buckling load and the lowest load at which buckling can occur in the presence of finite perturbations. The effect of imperfection geometry on the disturbance-sensitive load range, is shown in Figure 12.

In this plot, the circular marker represents an interpolated value at $\delta/t = 0.75$ and $\sigma/L = 0.05$, corresponding to a condition in which the critical buckling load falls below the minimum buckling load, resulting in the absence of the unstable bifurcation, as discussed previously. The results indicate that the size of this load interval is strongly influenced by both imperfection amplitude and width. For all amplitudes, the disturbance-sensitive range initially decreases with increasing imperfection width, reaches a minimum at an specific value of σ/L , and subsequently increases for larger widths. In addition, increasing the imperfection amplitude consistently reduces the disturbance-sensitive load range, indicating a progressive narrowing of the metastable regime as the severity of imperfections increases. These results demonstrate that geometric imperfections not only reduce the classical buckling load but also significantly alter the extent of the load interval over which the structure remains metastable, thereby directly affecting the sensitivity of the structure to disturbance-triggered buckling.

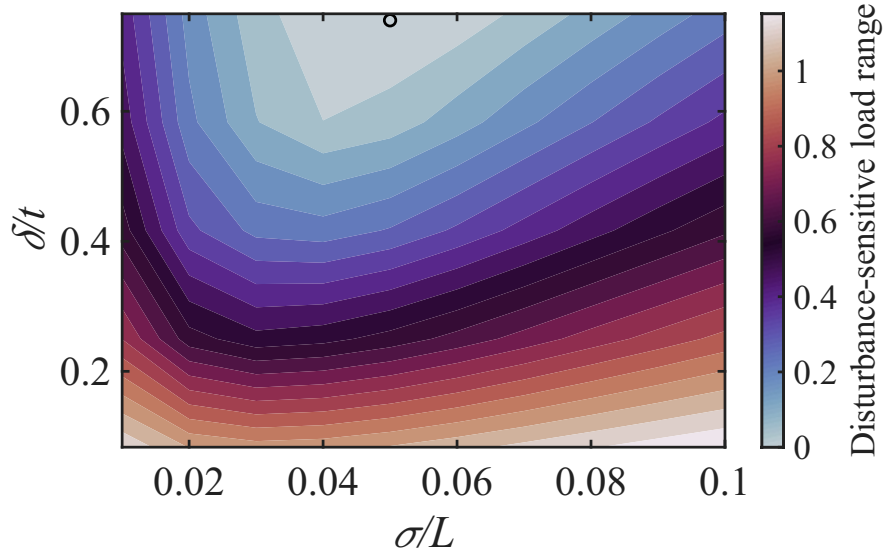


Fig. 12 Effect of inward imperfections on the disturbance-sensitive load range

V. Conclusion

This study investigated how localized geometric imperfections influence the buckling behavior of open-section thin-shell structures under bending, with emphasis on the buckling energy barrier as a quantitative measure of stability. Through finite-element probing simulations, the buckling initiation location, the role of dimple parameters, and the global disturbance-sensitive response were systematically evaluated.

Mapping the buckling energy barrier along the shell showed that the minimum barrier occurs at the symmetric mid-span, identifying it as the most instability-sensitive location. Regardless of the initial probing location, the post-instability response migrated to this coordinate, establishing it as the critical point at which imperfections have the greatest influence on buckling.

Under the specific applied moment, the effects of imperfection width and amplitude revealed the central role of the natural buckle width in governing stability. When the width of a positive dimple exceeded this threshold early buckling was suppressed, demonstrating that outward imperfections may be used intentionally to increase stability and raise the minimum buckling load. Negative dimples showed a dual behavior: for widths below the natural-buckle width, they unexpectedly increased stability by elevating the minimum buckling load, whereas wider dimples reversed this effect, lowering the energy barrier and promoting premature buckling.

Probing across the entire shell length showed that imperfections also reshape the disturbance-sensitive region. Positive dimples progressively localized this region as amplitude increased, constraining instability to a narrow spatial region and making buckle formation more deterministic. In contrast, negative dimples broadened this region and introduced a low-energy plateau, allowing buckling to occur over a wider range of locations and producing a more spatially unpredictable response.

Negative dimples significantly reduce the classical critical buckling load, whereas the minimum buckling load shows comparatively lower sensitivity and a non-monotonic dependence on imperfection width. The difference between these two loads defines the disturbance-sensitive load range. This range is strongly affected by imperfection geometry and determines the sensitivity of the structure to disturbance-triggered buckling.

Overall, these results demonstrate that local imperfections do not simply weaken thin shells; rather, they can be tuned to either stabilize or destabilize the structure depending on their geometry. The natural buckle width provides a governing scale that separates stabilizing and destabilizing regimes, while the disturbance-sensitive region reveals how imperfections influence both the severity and the spatial localization of buckling. These insights provide a pathway toward engineered imperfection tailoring for enhanced buckling resistance in ultra-lightweight structures. We are currently conducting experiments to validate these results.

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