

Predicting scar bistability in circular arcs corrugated sheets: An analytical, numerical, and experimental investigation

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Abstract

Multistable structures have been widely explored in aerospace engineering to develop innovative structures capable of shape-morphing and mechanical adaptivity. Among those, corrugated structures offer a highly effective approach to achieving bistability. In this paper, the bistability of circular arcs corrugated sheets with a scar, a local depression, is investigated using analytical models and a finite element (FE) model, which are then validated through experiments. Initially, a local simplified model is first developed to predict the longitudinal curvature at the center of a corrugated unit strip, demonstrating excellent agreement with FE results. Building on this, a global analytical model, based on the moment equilibrium equation and the Codazzi–Mainardi compatibility equations, is proposed to predict the evolution of both longitudinal and transverse curvatures throughout the entire geometry. The model further demonstrates that for an isotropic material, the scar-induced bistability is independent of material properties, confirming that the phenomenon is governed primarily by geometry. Using the framework, the required threshold aspect ratio for bistability is predicted as a function of the corrugation angle and the radius-to-thickness ratio. The model is also extended to investigate asymmetric corrugated unit strips, revealing the influence of geometric asymmetry on the bistable region and the resulting curvature distribution. Finally, the proposed framework is extended to study a multi-cell corrugated sheet, in which cell-cell interaction dominates. The

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results reveal that, even in multi-cell systems, the analytical model can effectively capture the curvature distribution over most of the corrugated sheet, demonstrating its applicability to both unit strip and multi-cell corrugated systems.

Keywords: Bistability, Corrugated sheet, Scar shape, Analytical model, Strain energy

1. Introduction

As aerospace structures become more advanced, the demand for shape-adaptability in novel autonomous systems grows, where the balance between stiffness and low mass often competes with the need for multifunctionality. Multistable structures have emerged as ideal candidates to provide mechanical adaptivity across a range of operational conditions without the need for continuous, thus mass-inefficient actuation. These systems are designed such that they exhibit multiple stable equilibria, encoded in their mechanical behavior, that can be accessed by applying an appropriate external perturbation to overcome the energy barrier between two chosen equilibria. This paradigm enables them to be used for diverse applications that involve shape-morphing aerospace structures, bioinspired structures, solar trackers and collectors, robotics, energy harvesting, microelectronics, and many others (Nicassio et al., 2018; Dong et al., 2023). On a mechanics standpoint, the associated nonlinear characteristics make their research even more engaging and, at the same time, challenging. Novel modeling and design techniques are required for the efficient static and dynamic analysis of these structures (Haldar et al., 2020).

Most multistable structures rely on bistable unit strips that harness snap-through instabilities to produce deformed stable states. Multistability arises from tessellating these building blocks in series and/or in parallel. Bistability is achieved through a diverse range of designs, each using distinct mechanical behaviors to maintain more than one stable configuration. Mansfield (1965) first demonstrated analytically bistability in an elliptical plate driven by a thermal gradient. The research additionally outlined the theoretical prerequisites for unperturbed systems to exhibit more than one stable state. Following, Hyer (1981) proposed bistability based on the thermal curing of unsymmetrical laminate. Building upon this contribution, a novel bistable

deployable structure of tubular shape, utilizing an antisymmetric laminate, was proposed by Iqbal et al. (2000), demonstrating stability in both the extended and rolled-up configurations (Galletly and Guest, 2004). Beyond composite layups, geometric nonlinearity is frequently exploited in the design of bistable systems using curved beams (Qiu et al., 2004), shallow arches (Liu et al., 2026), and von Mises trusses (De Castro et al., 2024), where external mechanical loading triggers a rapid transition across an energy barrier. Recently, these principles have been extended to advanced mechanical metamaterials (Shan et al., 2015), lattice structure (Pirrera et al., 2013) and origami-inspired architectures (Silverberg et al., 2014). Other strategies that have been employed to induce bistability included prestressing (Daynes et al., 2008; Suraj et al., 2026), initial curvature (Coburn et al., 2013), plastic deformation (Kebadze et al., 2004), and both discrete (Dai et al., 2012) and continuous structural connections (Luo et al., 2019).

Advanced morphing capabilities can also be achieved by embedding bistable elements in the corrugated structures. The prestressing approach has been extended to introduce bistability in corrugated sheets (Norman et al., 2008), where prestress is applied during manufacturing. The structure exhibits a primary stable state as a globally flat, stressed, corrugated sheet. However, these stresses are released by manually flattening the corrugations, and the sheet rapidly coils. A simplified analytical model was proposed to demonstrate this bistability, assuming material on two scales: the ‘local’ scale of the isotropic material and the ‘global’ scale of the corrugated sheet, which is modeled as an equivalent flat sheet with anisotropic properties. Having an initial curvature along the corrugations, the sheets can undergo significant elastic changes in shape, including large changes in overall Gaussian curvature and shape without any local stretching of the mid-surface (Norman et al., 2009). Walker and Seffen (2018) demonstrated bistability in ridge-like strip, highlighting that the bistable behavior can be achieved in a naturally non-flat strip without the need for prestressing. Subsequently, they investigated the influence of crease geometry on the large-displacement flexural behavior of thin creased strips (Walker and Seffen, 2019). Meeussen and Van Hecke (2023) introduced a specific class of corrugated sheet as a new shape-morphing mechanism that allows repeated switching from the flat state to multiple, precisely selected and stable 3D shapes. These sheets morph via stretching the sheet with point forces to introduce local defects in each groove. Once the defect forms, it self-organizes into a scar and be-

comes insensitive to perturbations and excess forcing. Later, Liu et al. (2025) introduced loading-position-dependent bistability in the same sheet, further demonstrating that localized perturbations provide a simple and effective route to achieving multistability in corrugated-sheet unit strips. A theoretical model was proposed based on energy minimization between bending and buckling deformations, in which the curvature equation for the imposed buckling deformation is derived from parametric finite element (FE) analyses. Further, the theoretical model was extended to predict the spatial extent of the bistable region for the multi-cell corrugated sheet. However, this proposed theoretical framework still requires parametric FE simulations to establish the curvature equation and utilizes the deformed curvature at the center as the primary state variable.

The present study proposes a generalized analytical model to accurately capture the scar, a local depression (Figure 1(d)), shape in a stress-free circular arcs corrugated unit strip over its entire geometry. The approach pairs a local description of deformation based on the widely studied tape spring problem (Jain and Rimrott, 1971; Seffen et al., 2019; Wang et al., 2021) with a higher-order shell model derived from thin and deep-shell theories and employing the Codaazzi-Mainardi compatibility conditions. This refined shell description provides an accurate account of the structure’s bistable behavior. It enables the extension of tape spring bistability studies to scenarios in which localization plays a major role, a feature the proposed model can effectively capture. The current unit strip geometry extends the conventional tape-spring geometry, with the surrounding corrugated sides providing the geometric constraint needed to stabilize the scar, which is absent in a conventional tape springs. Experimental investigations are also conducted using corrugated sheets fabricated from unidirectional glass fiber-reinforced polymer to demonstrate scar bistability. The frameworks are further extended to determine the minimum aspect ratio required for bistability across a wide range of corrugated geometries and to identify the combinations of geometric parameters that promote bistability, thereby providing practical design guidelines. Finally, the analytical framework is extended to investigate bistability in multi-cell corrugated structures, where the influence of cell-cell interactions on the curvature distribution and bistable response is examined. The proposed analytical model enables efficient parametric studies of both unit strip and multi-cell configurations without relying on difficult FE snap-through simulations.

This paper is organized as follows. Section 2 describes the geometry, the proposed analytical models, and the detailed derivation associated with it. It also presents the FE model used in the analyses. Section 3 explains the details of the experimental investigation. The analytical and numerical models are validated for predicting bistability and curvature evolution in composite corrugated sheets in Section 4. In Section 5, the analytical model is simplified for isotropic materials. Section 6 presents a parametric study to investigate the dependence of the bistable region on the geometric parameters. The framework is then extended to multi-cell corrugated structures to account for cell-cell interactions in Section 7. Finally, the main conclusions are summarized in Section 8.

2. Methods

2.1. Unit strip

The corrugated sheets investigated in this work consist of parallel wavy grooves, as shown in Figure 1(a). The x -axis is aligned along its length, and the y and z -axes are tangential and normal at any point on the corrugated sheet. As shown in Figure 1(b), the unit strip is divided into two distinct parts: Region *I* and Region *II*. The corresponding cross-section, illustrated in Figure 1(c), is composed of circular arcs with a uniform radius R_1 and R_2 in Region *I* and *II*, respectively. Section BC belongs to Region *I* and has a positive curvature, subtending an angle of $2\phi_1$ at its center. In contrast, Region *II* consists of segments AB and CD, which have opposite curvature. Together, these segments subtend a total angle of $2\phi_2$, with each segment subtending an angle of ϕ_2 at its respective center. Regions *I* and *II* are smoothly connected at their junctions, with the two regions being tangent at points B and C. This ensures continuity of the surface slope and provides a smooth transition between the two circular regions. Here, L and b denote the lengths of the corrugated sheet in the x - and y -directions, respectively. The thickness of the corrugated sheet is t . The structure is treated as a thin-walled shell, i.e., the thickness is significantly smaller than the overall dimensions of the sheet (i.e., $b, L \gg t$). Given this piecewise circular profile, the absolute curvature remains constant at $|\kappa_{y0}| = 1/(R_1, R_2)$. Still, the sign of the curvature κ_{y0} alternates between the regions. Additional geometric properties of the corrugated sheet are functions of the radii and corrugation angles (Suraj and Royer, 2026). Apart from Section 6.2, where the bistability

of the corrugated sheet is investigated by considering different geometries for Region *I* and Region *II*, the remaining analyses assume equal radii and angles for both regions, i.e., $(R_1 = R_2)$ and $(2\phi_1 = 2\phi_2)$.

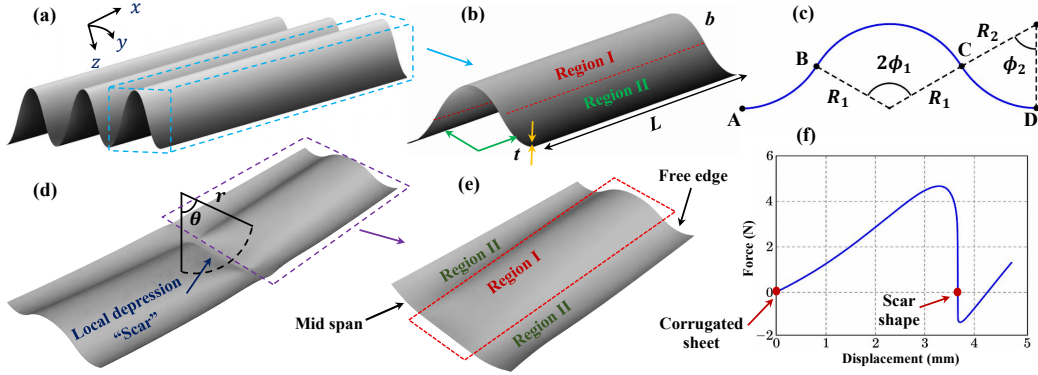


Figure 1: Schematic illustration and response of the corrugated sheet: (a) corrugated sheet with a circular cross-section, (b) representative unit strip, (c) cross-section of corrugated sheet, (d) scar configuration of the corresponding unit strip, (e) half-scar configuration showing localization of the scar within Region *I* while Regions *II* remain largely undeformed, and (f) representative load–deformation response of the composite corrugated sheet.

2.2. Local simplified model

The constitutive equations for a thin plate, in terms of stress resultants and mid-plane strains are:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} \quad (1)$$

where A , B , and D are the in-plane, coupling, and bending matrices of the laminate, respectively. The reader may refer to Thai and Choi (2013) for details of these matrices as a function of lamina properties and fiber angle.

Eq.1 can be rewritten in mixed form as:

$$\begin{bmatrix} \varepsilon \\ M \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ C^* & D^* \end{bmatrix} \begin{bmatrix} N \\ \kappa \end{bmatrix} \quad (2)$$

where

$$A^* = A^{-1}, \quad B^* = A^{-1}B, \quad C^* = BA^{-1}, \quad D^* = D - BA^{-1}B \quad (3)$$

For a symmetric laminate, $\mathbf{C}^* = \mathbf{0}$, and hence the bending moment is independent of the in-plane stress resultants. Although $\mathbf{C}^* \neq \mathbf{0}$ for the non-symmetric layup, its magnitude is negligible compared with that of $\mathbf{D}^*$ and hence the bending moment can be approximated solely as a function of the curvature:

$$\mathbf{M} = \mathbf{D}^* \boldsymbol{\kappa} \quad (4)$$

The local simplified model assumes that deformation is highly localized within Region *I*, while Region *II* experiences no change in curvature. These assumptions are based on experimental observations and were subsequently confirmed by FE analysis. Further, the stretching energy is ignored in both regions in the local simplified model. Under these assumptions, the idealized corrugated sheet behaves like a tape spring, with the primary distinction being that it undergoes highly localized deformation at its center. Within Region *I*, a uniform longitudinal ($\kappa_x = 1/r$) and zero transverse ($\kappa_y = 0$) curvatures are assumed in a central active portion subtending an angle θ in the x -direction, as shown in Figure 1(d). The deformation assumption holds strictly for an arc length of $r\theta$, and the region outside of this zone behaves identically to Region *II*, exhibiting zero strain and zero change in curvatures. Consequently, within this zone, the changes in longitudinal and transverse curvature are evaluated as κ_x and $-\kappa_{y0}$, respectively. Furthermore, we assume that there is no twisting curvature anywhere in the corrugated sheet. Thus, the bending strain energy over the fold region is:

$$U_b = \frac{b\theta}{\kappa_x} \begin{bmatrix} \Delta\kappa_x & \Delta\kappa_y \end{bmatrix} \begin{bmatrix} M_x \\ M_y \end{bmatrix} \quad (5)$$

Substituting the curvature change and Eq.4 in the above equation, and minimizing the total bending energy with respect to κ_x , yields the following expression for the curvature of the localized fold region (Yee et al., 2004):

$$\kappa_x = \sqrt{\frac{D_{22}^*}{D_{11}^*}} \kappa_{y0} \quad (6)$$

For an isotropic material, the above equation reduces to $\kappa_x = \kappa_{y0}$, commonly known as Rimrott's prediction (Rimrott, 1970). Seffen (2001) shows that the radius of the hinge in the tape-spring problem is larger than Rimrott's prediction by as much as 15% via experiment and FE analysis. The error is expected to be more pronounced in the present study, particularly with respect to the underlying assumptions regarding Region *II*.

2.3. Global analytical model

The local simplified model predicts the longitudinal curvature of the scar shape, assuming zero transverse curvature at the center. However, due to the highly localized deformation, the predicted curvature is valid only in a small region near the center. In this section, these insights are extended to develop a global analytical model that predicts the curvature field throughout the entire geometry.

2.3.1. Evolution of transverse curvature (κ_y)

To analytically capture the complex transition of the corrugated shell from a flat, stress-free configuration to a localized deformed state, a curvature approximation is employed using Donnell's shell theory and Codazzi–Mainardi compatibility equations. Under the established assumption that the curvature in Region *II* remains constant, the Region *I* of the corrugated sheet can be idealized as Donnell's cylinder. The out-of-plane equilibrium for bending for Region *I* (assuming no external transverse pressure) is:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = 0 \quad (7)$$

Using Eq.4, the above equilibrium equation modified to:

$$D_{11}^* \frac{\partial^2 \kappa_x}{\partial x^2} + D_{12}^* \frac{\partial^2 \kappa_y}{\partial x^2} + 2D_{66}^* \frac{\partial^2 \kappa_{xy}}{\partial x \partial y} + D_{12}^* \frac{\partial^2 \kappa_x}{\partial y^2} + D_{22}^* \frac{\partial^2 \kappa_y}{\partial y^2} = 0 \quad (8)$$

Using the Codazzi–Mainardi compatibility equations, the mixed partial derivatives of curvature, i.e, $\partial^2 \kappa_{xy} / \partial x \partial y$ can be expressed either as $\partial^2 \kappa_x / \partial y^2$ or $\partial^2 \kappa_y / \partial x^2$. By splitting the mixed partial derivatives into one of each, the moment equilibrium equation can be written in term of curvature derivatives as:

$$D_{11}^* \frac{\partial^2 \kappa_x}{\partial x^2} + (D_{12}^* + D_{66}^*) \frac{\partial^2 \kappa_x}{\partial y^2} + (D_{12}^* + D_{66}^*) \frac{\partial^2 \kappa_y}{\partial x^2} + D_{22}^* \frac{\partial^2 \kappa_y}{\partial y^2} = 0 \quad (9)$$

Following the work of Jain and Rimrott (1971) and Seffen et al. (2019) on curvature evolution in tape-springs, the moment equilibrium is significantly simplified by assuming that the second derivative of the transverse curvature dominates, as per Donnell, and the rate of change of longitudinal curvature along y is some fixed proportion, α , of the rate of change of the transverse

curvature in the same direction. By substituting this relationship into the equilibrium equation, a final governing equation for κ_y is obtained as:

$$\frac{\partial^2 \kappa_y}{\partial x^2} + \lambda^2 \frac{\partial^2 \kappa_y}{\partial y^2} = 0, \quad \text{where} \quad \lambda = \sqrt{\frac{D_{22}^*}{D_{12}^* + D_{66}^*} + \alpha} \quad (10)$$

where λ is a growth parameter for transverse curvature. It should be noted that the above equation is valid only if all components of the modified bending matrix ($\mathbf{D}^*$) are of the same order of magnitude. If D_{11}^* is significantly larger than the other terms, the product $D_{11}^* \partial^2 \kappa_x / \partial x^2$ cannot be ignored, even if the $\partial^2 \kappa_x / \partial x^2$ is not dominating.

Jain and Rimrott (1971) employed a Fourier series expansion subject to appropriate boundary conditions, to solve for κ_y in Eq.10. Later, Seffen et al. (2019) proposed a variable separable solution of the form $\kappa_y = f(x) \cos(\pi y/b) + A$, where A is a constant. Additionally, Liu et al. (2023) developed a similar governing equation to investigate the folding behavior of a thin-walled tubular deployable composite boom. Adopting the approach of Seffen et al. (2019), and solving Eq.10 yields the following expression for κ_y :

$$\kappa_y = A + B \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right) \quad (11)$$

where constants A and B need to be found using the boundary conditions of the deformed shape.

As illustrated in Figure 1(e), the physical boundary conditions of half-scar configuration dictate that one end of Region *I* is completely flattened, while the opposite end (“infinity” for simplicity) retains its fully corrugated, unstressed profile. By applying these specific boundary conditions to the derived solution for κ_y , the governing transverse curvature in Region *I* can be expressed as:

$$\kappa_y = \kappa_{y0} - \kappa_{y0} \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right) \quad (12)$$

which complies with the curvature approximation at the center made in the local simplified model and recovers the undeformed curvature far away from the center.

The twisting curvature can be derived from the Codazzi–Mainardi compatibility equation (Eq.B.2), with the assumption of a zero twisting boundary condition along the centerline. However, the analyses indicate that the twisting curvature is negligible.

2.3.2. Localized longitudinal curvature (κ_x)

Following the derivation of the transverse and twisting curvatures, the longitudinal curvature can likewise be obtained from the Codazzi–Mainardi equations. It is important to note, however, that these relations are valid only within the framework of shallow shell theory. In contrast, scar configurations are stable in the intermediate-to-deep shell regime. As demonstrated in Appendix B, deriving all three curvature components using shallow shell theory does not accurately predict the scar shape. Consequently, a deep-shell formulation is employed in the present section to derive the longitudinal curvature relation. Thus, the thin-shell theory of Novozhilov, which applies to shells of arbitrary depth, is adopted for the derivation of localized longitudinal curvature. The transverse and longitudinal membrane strain and changes in curvatures are related to the displacement field as follows (Novozhilov, 1999):

$$\begin{aligned}\epsilon_x &= \frac{\partial u}{\partial x}, & \epsilon_y &= \frac{\partial v}{\partial y} - \kappa_{y0} w \\ \kappa_x &= -\frac{\partial^2 w}{\partial x^2}, & \kappa_y &= -\frac{\partial^2 w}{\partial y^2} - \kappa_{y0} \frac{\partial v}{\partial y}\end{aligned}\tag{13}$$

Assuming inextensional transverse deformation ($\epsilon_y \approx 0$) implies $\partial v / \partial y \approx \kappa_{y0} w$ and neglecting twisting curvature, the curvature changes are related to the out-of-plane displacement as:

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2} - \kappa_{y0}^2 w\tag{14}$$

Assuming out-of-plane displacement exists and partial derivatives are continuous, this allows the order of differentiation to be interchanged. Differentiating the curvatures κ_x and κ_y twice with respect to y and x , respectively, and rearranging the terms yields the compatibility equation for an arbitrary shell as:

$$\frac{\partial^2 \kappa_x}{\partial y^2} + \kappa_{y0}^2 \kappa_x = \frac{\partial^2 \kappa_y}{\partial x^2}\tag{15}$$

The right-hand side of the above equation can be evaluated using Eq.12. Consequently, the general solution, comprising both the homogeneous and particular parts, is derived as:

$$\kappa_x = c_1(x) \cos(\kappa_{y0} y) + c_2(x) \sin(\kappa_{y0} y) - \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right)$$

(16)

where $c_1(x)$ and $c_2(x)$ are integration constants. Due to the symmetry of the problem about the y -axis, $c_2(x)$ must be zero. This assumption, together with the choice of the trial function for the transverse curvature, limits the applicability of the analytical model to scar shapes that are symmetric about the y -direction. The remaining constant, $c_1(x)$, is then determined by matching the longitudinal curvature along $y = 0$ with observations from the local simplified model, FE simulations and experiments. The local simplified model predicts that $\kappa_x = \sqrt{D_{22}^*/D_{11}^*} \kappa_{y0}$ at the center, while FE and experiment observations indicate that this curvature decays almost instantaneously, governed by a decay parameter ξ . Thus, the longitudinal curvature along x at $y = 0$ can be approximated as $\sqrt{D_{22}^*/D_{11}^*} \kappa_{y0} \exp(-\xi x/L)$ and upon substituting in Eq.16, the unknowns $c_1(x)$ can be expressed as:

$$c_1(x) = \sqrt{\frac{D_{22}^*}{D_{11}^*}} \kappa_{y0} \exp\left(-\xi \frac{x}{L}\right) + \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \quad (17)$$

where ξ is the decay parameter, which is discussed in detail in Appendix A. This parameter depends solely on the aspect ratio ($AR = L/b$), allowing it to be treated as a constant for any fixed aspect ratio (Figure A.19). Substituting $c_1(x)$ in Eq.16 gives the expression for the longitudinal curvature as:

$$\begin{aligned} \kappa_x = & \left(\sqrt{\frac{D_{22}^*}{D_{11}^*}} \kappa_{y0} \exp\left(-\xi \frac{x}{L}\right) + \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \right) \cos(\kappa_{y0} y) \\ & - \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right) \end{aligned} \quad (18)$$

It is important to note that both curvature equations are independent of material properties, except for the boundary condition for longitudinal curvature at $y = 0$ used to determine the integration constant.

2.3.3. Bending and stretching energy formulation

Jain and Rimrott (1971) acknowledge that because the bending solution foists a change in Gaussian curvature, there is also in-plane stretching primarily along the x -direction, coinciding with the Donnell's cylinder's axis. Therefore, to ensure an admissible displacement field, the curvature and membrane

strain are governed by the Gauss compatibility condition, expressed as:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} + \frac{\partial^2 \epsilon_y}{\partial x^2} = \kappa_{xy}^2 - \kappa_x \kappa_y \quad (19)$$

With the assumptions of negligible shear strain, transverse strain, and twisting curvature, i.e., $\gamma_{xy} \approx 0$, $\epsilon_y \approx 0$ (length b assumed to be the same after deformation), and $\kappa_{xy} \approx 0$, respectively, the curvature strain compatibility equation simplifies to:

$$\frac{\partial^2 \epsilon_x}{\partial y^2} = -\kappa_x \kappa_y \quad (20)$$

Using the above compatibility equation, the strain component ϵ_x in Region I and Region II can be expressed following the symmetry argument as:

$$\begin{aligned} \epsilon_{xI} &= - \int \left(\int \kappa_x \kappa_y dy \right) dy + c_4(x) \\ \epsilon_{xII} &= c_6(x) \end{aligned} \quad (21)$$

where $c_4(x)$ and $c_6(x)$ are integration constants. It is important to note that both κ_x and κ_y are functions of x and y . The constants, $c_4(x)$ and $c_6(x)$, are evaluated by applying two conditions: boundary strain compatibility between Region I and Region II and net zero axial force in the y -direction for any given cross-section.

Evaluating these unknowns yields the full explicit expressions for the strains. Furthermore, from Eq.21, the axial strain, ϵ_x , is independent of the transverse coordinate y in Region II . With the strain field fully resolved, the total stretching strain energy of the system can subsequently be formulated. Since the change in curvature in Region II is assumed to be zero, the stretching energy in this region depends solely on the stretching term A_{11} . Conversely, the stretching energy of Region I depends on both the strain and change in curvature due to coupling terms, expressed as:

$$\begin{aligned} U_{sI} &= \frac{1}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} [A_{11} \epsilon_{xI}^2 + B_{11} \kappa_x^2 + 2 B_{12} \kappa_x (\kappa_y - \kappa_{y0}) + B_{22} (\kappa_y - \kappa_{y0})^2] dy dx \\ U_{sII} &= \frac{1}{2} \frac{b}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} A_{11} \epsilon_{xII}^2 dx \end{aligned}$$

(22)

Similarly, the bending energy of these regions is expressed as:

$$\begin{aligned}
 U_{b_I} &= \frac{1}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} [B_{11}\epsilon_{x_I}^2 + D_{11}\kappa_x^2 + 2D_{12}\kappa_x(\kappa_y - \kappa_{y0}) + D_{22}(\kappa_y - \kappa_{y0})^2] dy dx \\
 U_{b_{II}} &= \frac{1}{2} \frac{b}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} B_{11}\epsilon_{x_{II}}^2 dx
 \end{aligned}
 \tag{23}$$

Thus, the total strain energy is solely function of λ as:

$$U(\lambda) = U_{s_I}(\lambda) + U_{s_{II}}(\lambda) + U_{b_I}(\lambda) + U_{b_{II}}(\lambda) \tag{24}$$

Due to the complexity of the analytical expressions for the curvatures, a numerical Gauss quadrature method is implemented in MATLAB to perform the integration, and a convergence study is conducted with respect to the number of Gauss points. Following the minimum strain energy theorem, the equilibrium equation is derived from the stationarity condition for the total strain energy and subsequently solved for λ using the `fminunc` solver routine in MATLAB.

2.4. Finite Element model

The FE analysis of corrugated sheets has been performed considering geometric nonlinearities in a commercial general-purpose FE package, `ABAQUS`. Shell elements of type S4R (4-node quadrilateral shell element with reduced integration) with a typical mesh size of 35 elements in the curvilinear coordinate y -direction are chosen to carry out the FE analysis. The mesh size is based on a tradeoff between computational time and solution accuracy. A flowchart detailing the simulation process is presented in Figure 2.

The geometric and material properties of the corrugated sheet are first defined. Two different material models are considered in the FE analyses: an isotropic linear elastic model and an orthotropic linear elastic model for the composite lamina. The sheet has a uniform cross-section along the x -direction, with its overall length being a real multiple of the corrugation wavelength. Many researchers have used a uniform moment in the folding of tape spring (Seffen and Pellegrino, 1999; Yee et al., 2004; Seffen et al., 2019), which works very well for corrugated sheets with small corrugation angles;

however, for large corrugation angles, local deformations at the points of application of the moment dominate and prevent the triggering of the snap-through instability. Hence, in the present study, we have adopted a local point displacement-based perturbation in the FE analyses.

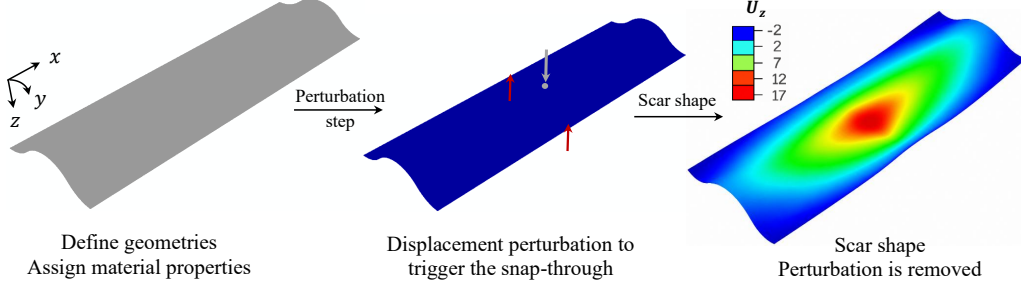


Figure 2: Schematic representation of the finite element framework. Contours are out-of-plane displacement.

In a first static step, a transverse displacement is applied at the center node of the geometry to induce snap-through with all the other degrees of freedom remaining restrained. The corresponding nodes at the center of both edges are restrained in the y -direction but remain free to rotate. The displacement applied to the corrugated sheet should be sufficient to trigger the snap-through instability, forming a stable scar shape. The snap-through process is simulated using the ABAQUS static general solver, employing automatic stabilization with a specified fixed dissipated energy fraction of 0.0002 to achieve a converged equilibrium solution. Once the full snap-through has occurred, the applied displacement is removed, allowing the sheet to evolve toward its new stable configuration. As in the loading phase, automatic stabilization is maintained during unloading to ensure numerical convergence. A typical force–deformation response obtained by following the above-mentioned boundary conditions and stabilization technique is shown in Figure 1(f) for the composite corrugated sheet. It highlights that the second deformed equilibrium is, indeed, load-free and stable. Note that the load displacement characteristic is obtained under displacement control and that kinetic energy is realized following the vertical tangent caused by the snap-through instability. This energy is dissipated by viscous damping here, but an equilibrium path may be obtained by using an arclength solver (Riks method for instance).

3. Experimental investigation

This section provides details on the composite manufacturing process employed to manufacture corrugated unit strips. While these structures are traditionally obtained by plastically deforming metal sheets, springback resulting from the material’s elastic recovery is a major cause of geometric inaccuracy, and residual stresses can be hard to account for in experiments. Thus, in the present study, the corrugated sheets are manufactured using unidirectional glass-fiber reinforced polymer (UD-GFRP, S2/PMT-F7 from Patz Materials and Technologies). The material properties are summarized in Table 1. Although the analytical model accounts for bending-stretching coupling, the composite laminate is assumed to have a symmetric stacking sequence, $[90^\circ/0^\circ/90^\circ]$, to eliminate it and simplify the analysis. However, the overall effects of these couplings are qualitatively examined in Section 4.4.

Table 1: Material properties of the UD-GFRP lamina (Abayneh et al., 2026).

Material	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	t (μm)
GFRP	55.0	5.5	6.4	0.26	55

An enclosed silicone-molding process has been adopted to produce composite corrugated sheets, using positive and negative silicone rubber molds (Smooth-on mold Max XLS II) constrained within aluminum cages (Royer and Pellegrino, 2018; Schlothauer et al., 2018). The manufacturing process is based on the thermal expansion difference between silicone rubber and aluminum, as well as silicone rubber’s quasi-incompressibility, which allows high pressure to be applied to the layup without changing the cross-sectional shape. Although an autoclave-curing method with rigid molds has traditionally been used (Salazar and Fernandez, 2021), it incurs significantly higher capital and operational costs than oven-based processing. The current manufacturing method provides a rapid, cost-effective, and accurate way to cure autoclave composites in an out-of-autoclave process.

A 3D-printed polylactide (PLA) mold negative is used to fabricate a two-part silicone mold within an aluminum cage through sequential casting, as shown in Figure 3. The first half mold is cast around the 3D-printed part and then cured. After removing the printed part, the cured half is retained in the cage, coated with a thin layer of release agent (Mann Ease Release 200),

and used as the base for casting the second half. Then, the desired composite plies are carefully positioned between the positive and negative silicone mold halves. The assembled mold, containing the composite laminate, is then enclosed within a rigid aluminum cage. During the high-temperature curing cycle, the mismatch in thermal expansion between the silicone mold and the aluminum cage generates substantial internal hydrostatic pressure, ensuring effective consolidation of the composite laminate. This process ensured accurate alignment and smooth mating surfaces between the two mold halves. After oven curing, the corrugation is perturbed by point load to achieve the scar shape. Then the specimens are scanned using a 3D scanner (Shining 3D FreeScan UE Pro) to generate 3D point clouds that define the corrugated and scar surfaces. Each specimen is placed on a reference plate fitted with scanning markers and captured from multiple viewing angles. The acquired point clouds are then processed and visualized using the **FreeScan** software.

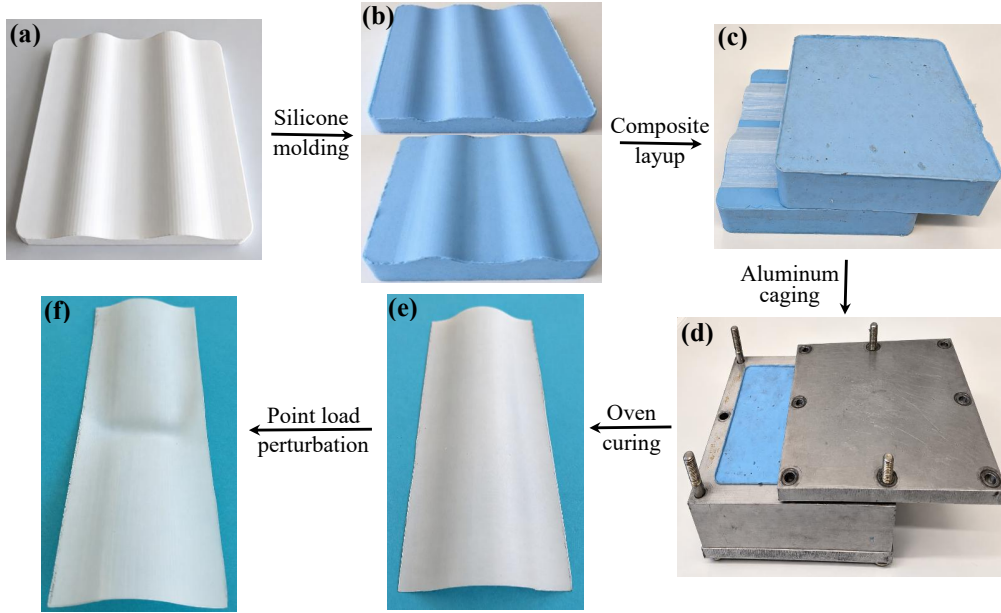


Figure 3: Experimental fabrication procedure for the composite corrugated sheet. (a) PLA mold of the prescribed geometry; (b) corresponding positive and negative silicone molds; (c) composite laminate enclosed between the molds; (d) mold assembly confined in an aluminum cage during curing; (e) cured corrugated composite specimen; and (f) localized scar formed by a central point-load perturbation.

4. Validation

This section presents the validation of the FE model by comparing it with the experimental results. For simulations, as a case study, three different corrugated sheet geometries are considered, with radius $R = 20\text{mm}$, corrugation angles $2\phi_0 = 45^\circ$, 60° , and 75° and lamina stacking sequence of $[90^\circ/0^\circ/90^\circ]$.

4.1. Deformed shape

Figure 4 presents the experimentally obtained corrugated sheets and their corresponding scar configurations for different corrugation angles. Since the aluminum cage has a fixed length of 131mm, the aspect ratio of the corrugated sheet varies with the corrugation angle, 4, 3, and 2.5 for $2\phi_0 = 45^\circ$, 60° , and 75° , respectively. The corrugated sheets are carefully perturbed at the center to achieve the scar shape. Further, applying a suitable strong flattening force erases the scar pattern and returns the sheet to its initial corrugated state. The aluminum cage cap developed a slight longitudinal curvature due to pressure from the silicone mold during curing. Consequently, the manufactured sheets also have an imperceptible initial longitudinal curvature.

Figure 5 present the FE and scanned deformed configurations of the half-scar shape along with the corresponding mid-span and end cross-sections for all considered geometries. Since the deformation is symmetric about the center, only one half of each 3D-scanned specimen is compared with the FE prediction to improve visualization. The experimental cross-sections are represented by a smoothed mid-plane curve obtained from the point cloud data. The end cross-section is also shifted slightly to facilitate better comparison of cross-sections with the corresponding FE prediction. Overall, the experimentally measured deformation agrees well with the FE results. A small discrepancy is observed near the mid-span because the out-of-plane displacement is constrained in the FE model, whereas the experimental specimen is free to move. The experimental shapes also exhibit slight twisting, which may result from minor misalignment of the laminae during fabrication. Additional sources of error may include measurement uncertainty in the 3D scanning process and errors introduced during the acquisition and processing of the scanned geometry. Nonetheless, the results show good agreement between FE and the experiment. Upon validation against the experiment, the FE model serves as a reference for the remaining sections.

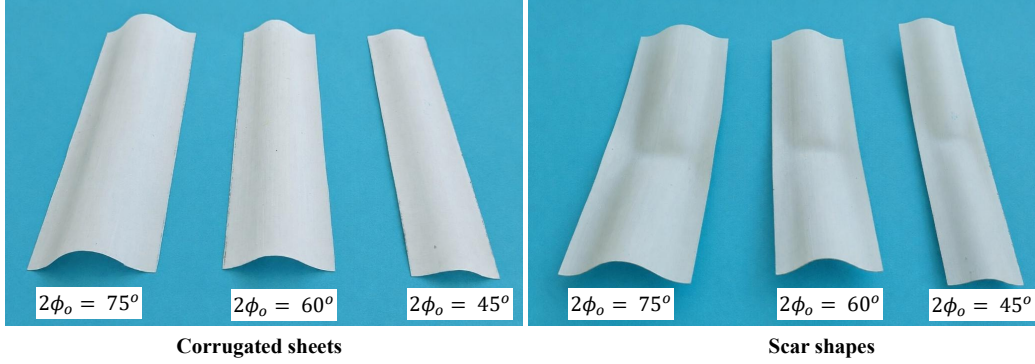


Figure 4: Experimentally obtained corrugated sheets and corresponding scar shapes for different corrugation angles.

4.2. Local simplified model

Figure 6 compares the curvatures predicted by the FE and local simplified model at the cross-section $x = 0$. In the FE analysis, the curvatures are computed from the deformed nodal coordinates by locally fitting a circle to the deformed profile. The simplified model provides a reasonable approximation of both longitudinal and transverse curvatures at the center. Although the transverse curvature approximation is relatively accurate, the longitudinal curvature results vary by about 15-25% when compared to the FE results for all corrugation angles. This discrepancy aligns with the findings of Seffen (2001), in which the analytically predicted curvature varies by about 15% compared to experiment and FE analysis. The larger discrepancy observed between the FE results and the simplified analytical model in the present study is primarily due to neglecting the energy contribution from Region *II* and idealizing the corrugated sheet as a tape spring. Furthermore, the applicability of the simplified model is limited to a small region near the center, due to highly localized deformation observed in FE and experimental analyses. This limitation of the local simplified model is effectively addressed by the global analytical model, which accurately captures curvature variation along the length.

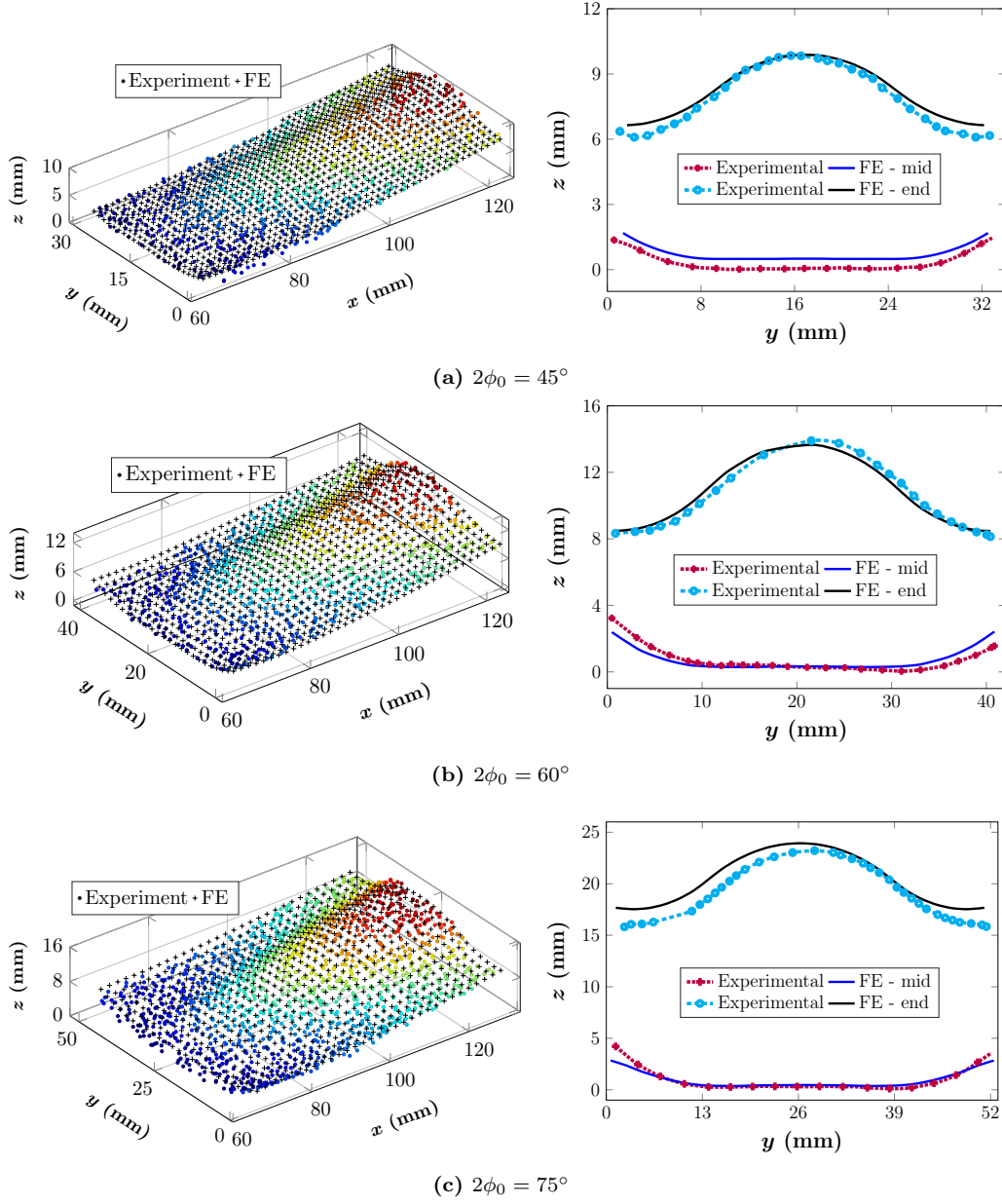


Figure 5: Comparison of experimentally observed and numerically predicted half-scar shapes and corresponding mid- and end-section profiles for different corrugation angles.

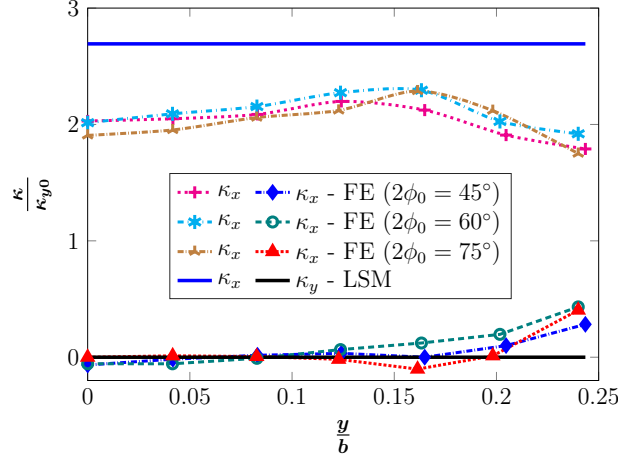


Figure 6: Transverse and longitudinal curvatures prediction along the cross-section at $x = 0$ using the local simplified model and the FE analysis.

4.3. Global analytical model

While the local simplified model adequately approximates the central curvature, it is unable to predict the curvature over the rest of the geometry. However, the proposed global analytical model can predict the curvature over the entire geometry. As shown in Figure 1(e), the scar shape exhibits symmetry; therefore, the half-length curvature derived from the global analytical model is plotted alongside the FE result.

Figure 7 illustrates the curvature distribution for the scar shape along the x and y -directions, for the cross-section at $y = 0$ and $x = 0$, respectively, in the composite corrugated sheets. The transverse curvature exhibits an increasing trend along the length, whereas the longitudinal curvature decays rapidly along the x -direction. The analytical model aligns closely with the numerical results, except for the longitudinal curvature at the center. An approximate 20% difference in longitudinal curvature at the center is observed for all corrugation angles. This variation overlaps with the analytical deviations noted by Seffen (2001) in standard tape-spring models. The transverse curvature remains zero near the center within a finite region. In contrast, the longitudinal curvature decays rapidly from the center, reverses its sign relative to the undeformed configuration, and then gradually approaches zero over the remaining length. A small decay/oscillation near the origin is observed, which may be due to local buckling, which has not been accounted for in the analytical formulation. Again, Calladine and Seffen (2020) reported

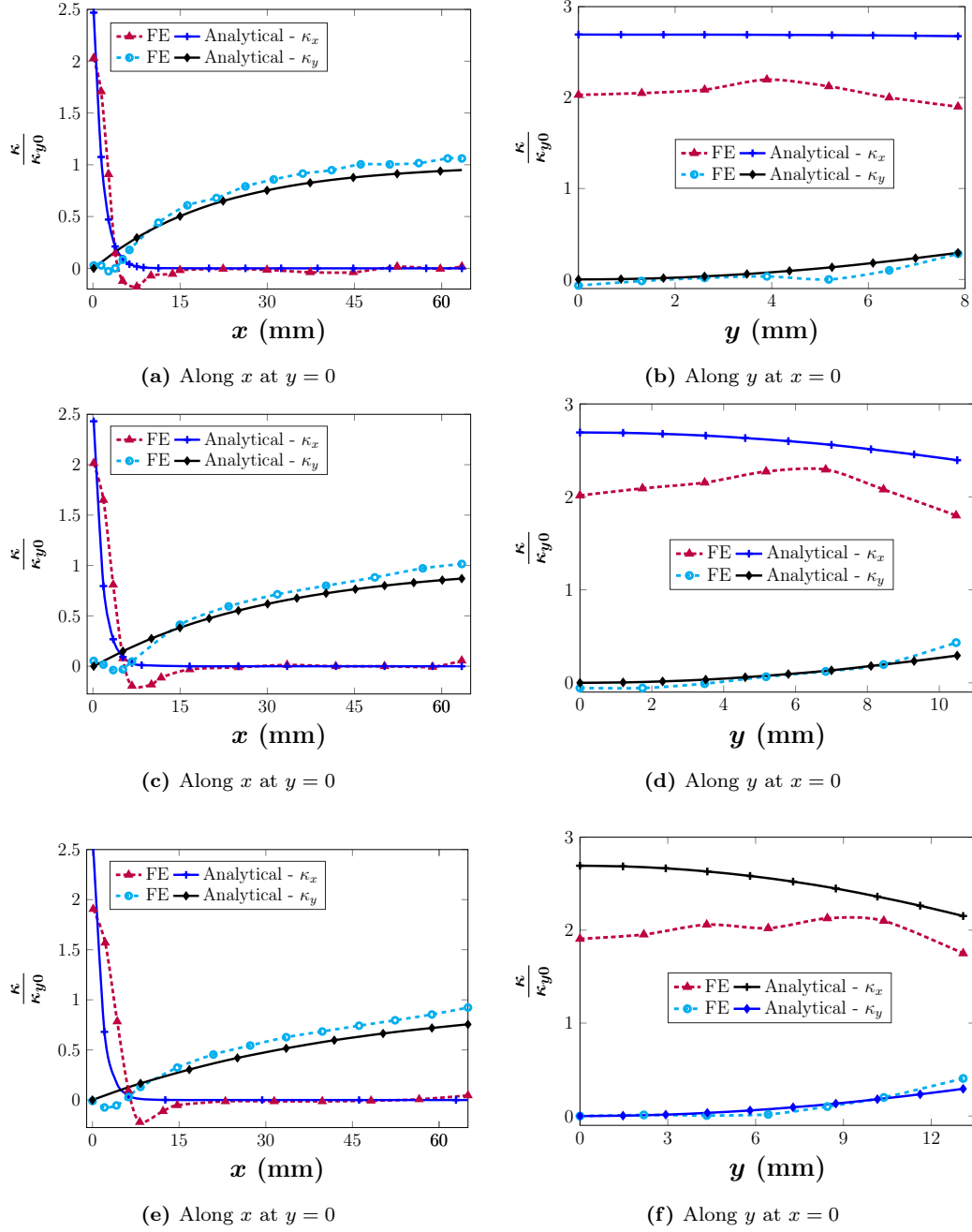


Figure 7: Transverse and longitudinal curvatures of the scar shape in GFRP corrugated sheets along the centerlines for different corrugation angles: (a,b) $2\phi_0 = 45^\circ$, (c,d) $2\phi_0 = 60^\circ$, and (e,f) $2\phi_0 = 75^\circ$.

a similar pattern of curvature in the context of the tape-spring problem. The longitudinal curvature along the y -direction shows a similar trend, with an approximately 20% difference at the center and a reduction at the end of Region *I*. However, the analytical prediction of transverse curvature along y closely follows the FE result for all corrugation angles.

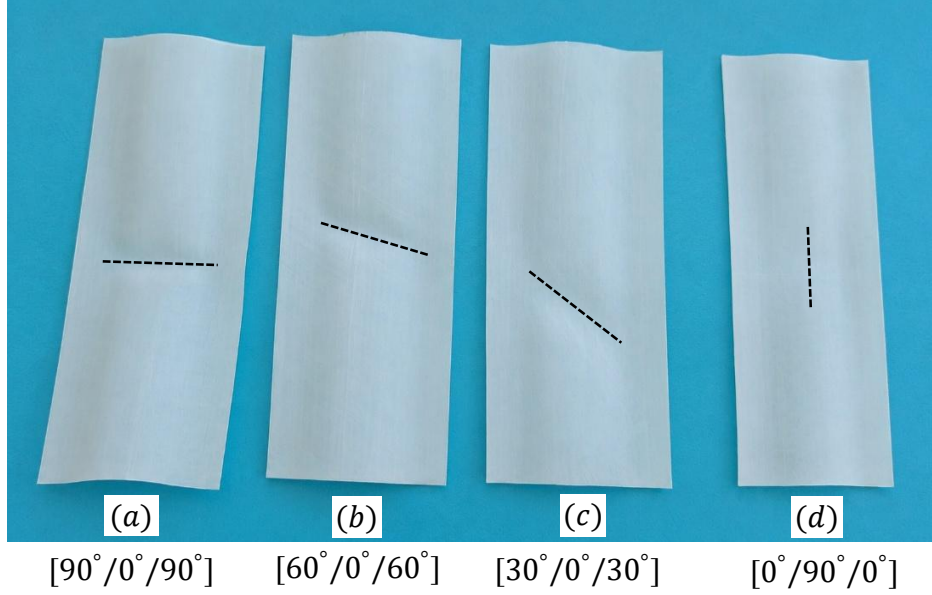


Figure 8: Scar orientation in corrugated sheets with varying fiber angles of lamina.

4.4. Varying fiber angles

The stretching, bending, and bending–stretching coupling matrices of a composite laminate vary with changes in material properties and fiber orientations. Variation in the fiber angle alters the laminate’s anisotropic response, leading to changes in the resulting scar shape. Figure 8 shows the experimentally obtained scar shapes for different lamina fiber orientations. The laminates in Figure 8(b) and (c) exhibit non-zero B_{16} and B_{26} components in the bending–stretching coupling matrix, introducing extensional–twist coupling. These coupling effects modify the preferred deformation direction and dictate the scar orientation. Additionally, the normalized longitudinal curvature at the center (Eq.6) varies from 2.49 for the $[90^\circ/0^\circ/90^\circ]$ laminate to 0.37 for the $[0^\circ/90^\circ/0^\circ]$ laminate, as shown in Figure 8. The higher longi-

tudinal curvature in the $[90^\circ/0^\circ/90^\circ]$ laminate results in a more stable scar configuration compared to the $[0^\circ/90^\circ/0^\circ]$ laminate.

5. Parametric study of isotropic corrugated sheets

In this section, an isotropic material with Young's Modulus E and Poisson's ratio ν is considered to provide better insight into the bistability behavior of the corrugated sheet. Under this assumption, the curvature equations are reduced to:

$$\begin{aligned}\kappa_y &= \kappa_{y0} - \kappa_{y0} \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right) \\ \kappa_x &= \left(\kappa_{y0} \exp\left(-\xi \frac{x}{L}\right) + \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right)\right) \cos(\kappa_{y0} y) \\ &\quad - \frac{\lambda^2}{\kappa_{y0}} \left(\frac{\pi}{b}\right)^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \cos\left(\frac{\pi y}{b}\right)\end{aligned}\quad (25)$$

Similarly, the bending, stretching and total strain energy of the system reduces to:

$$\begin{aligned}U_{bI} &= \frac{D}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} [\kappa_x^2 + 2\kappa_x(\kappa_y - \kappa_{y0}) + (\kappa_y - \kappa_{y0})^2] dy dx, \quad U_{bII} = 0 \\ U_{sI} &= \frac{A}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} \epsilon_{xI}^2 dy dx, \quad U_{sII} = \frac{A}{2} \frac{b}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \epsilon_{xII}^2 dx \\ U(\lambda) &= U_{sI}(\lambda) + U_{sII}(\lambda) + U_{bI}(\lambda)\end{aligned}\quad (26)$$

where $A = Et/(1 - \nu^2)$ and $D = Et^3/12(1 - \nu^2)$ represent the extensional and flexural stiffness of an isotropic shell, respectively. Both the extensional and flexural stiffness matrices exhibit the same functional dependence on the material parameters. Therefore, this common material dependence cancels out in the minimization of the total strain energy, making the resulting equilibrium solution for λ independent of the material properties.

5.1. Analytical prediction

A Beryllium Copper (Be-Cu) alloy is chosen, as an isotropic material, with a Young's modulus of $E = 130\text{GPa}$ and a Poisson's ratio of $\nu = 0.3$ (Norman et al., 2007). A circular arcs unit strip with an initial radius $R = 20\text{mm}$,

subtended angle of $2\phi_0 = 45^\circ$, 60° , and 75° , a uniform thickness $t = 0.125\text{mm}$ and an aspect ratio of 5 is chosen for the analysis as an example. Hereafter, the global analytical model is referred to simply as the analytical model.

Figure 9 presents the analytical predictions of the curvature distribution over the entire Region *I* of corrugated sheet for an opening angle of 45° . The transverse curvature exhibits an increasing trend in both the x - and y -directions, whereas the longitudinal curvature remains nearly constant along the y -direction and decays rapidly along the x -direction. However, as with composite corrugated sheets, a slight decay in longitudinal curvature in the y -direction is also observed at higher opening angles (Figure 7(d) and (f)).

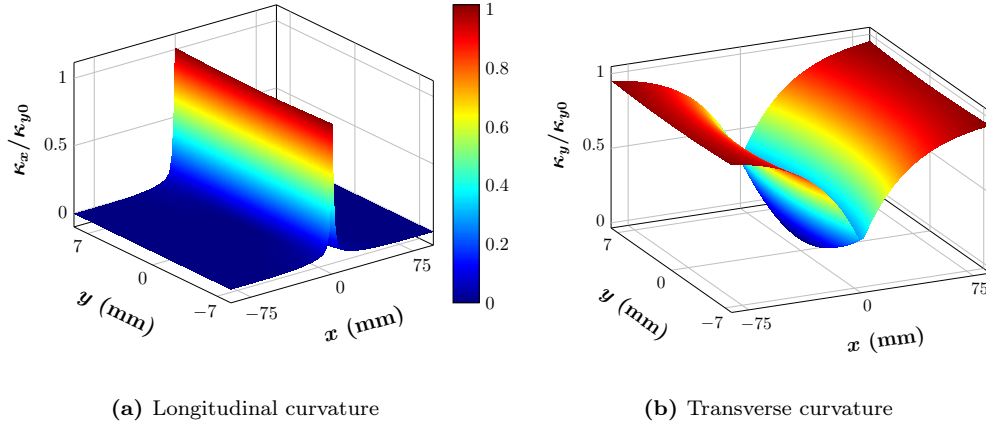


Figure 9: Analytical curvatures prediction over the entire Region *I* for $2\phi_0 = 45^\circ$.

The analytical predictions are subsequently validated against FE results for the $y = 0$ cross-section, as shown in Figure 10. For $2\phi_0 = 45^\circ$ (Figure 10(a)), the analytical model results align closely with those obtained numerically. Similar to the composite corrugated sheet, an approximately 20% difference in longitudinal curvature is observed at the center. The FE prediction indicates that the corrugated sheet is flat near the center in the x -direction. In contrast, the analytical model cannot capture this local variation because it assumes that the change in curvature is zero at the center and that the curvature grows exponentially. This discrepancy closely resembles the central deviation reported in the orthotropic tape-spring analysis of Seffen et al. (2019). Additionally, slight variations near the sheet end are also observed due to free-boundary effects. Past studies, such as Boddapati et al. (2024), accounted for similar boundary-induced errors in prestressed lami-

mates by selectively truncating the outer regions. However, in this study, the predicted curvatures are plotted across the full geometry for completeness. The curvatures variation along the y -direction is similar to GFRP composites, except that the normalized longitudinal curvature at the center is 1 for isotropic material.

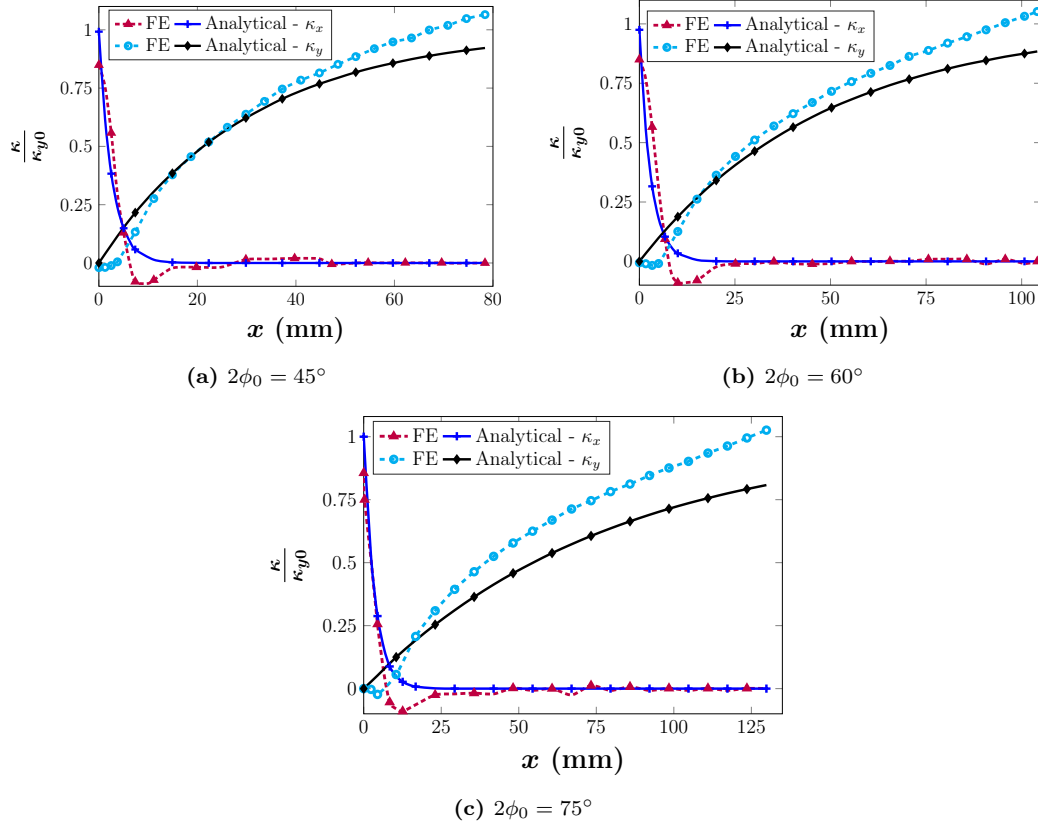


Figure 10: Transverse and longitudinal curvatures of scar shape in isotropic corrugated sheet along the centerline ($y = 0$) with varying corrugation angle.

Figures 10(b) and (c) depict the curvature distributions predicted by the FE and analytical model for corrugation angles of $2\phi_0 = 60^\circ$ and 75° , respectively. For the 60° case, the models show good agreement, with slight deviations similar to those observed in the 45° configuration. However, at $2\phi_0 = 75^\circ$, a more significant discrepancy is noticed in the dominant transverse curvature. Overall, the analytical model shows good agreement with the trend in curvature growth. However, it predicts a slightly slower rate of

variation of transverse curvature than observed in the FE analysis, which is treated as the reference solution. This difference may be attributed to the assumption of zero ϵ_y in the model, especially at the center. Conversely, in the FE simulations, these strains are small but not negligible. Another possibility is the theory underlying the derivation of transverse curvature, which applies only to shallow shells, although longitudinal curvature is derived using deep shell theory. Seffen et al. (2019) reported a similar divergence at larger opening angles in their study of orthotropic tape springs.

Figure 11 compares the strain energy predicted by the FE and the analytical models. Consistent with the trends observed in the curvature profile, the analytical model accurately predicts the strain energy associated with the scar shape for the smaller corrugation angle of 45° and provides a good approximation for the 60° case. However, the discrepancy increases with corrugation angle, reaching approximately 30% at 75° . The deeper energy well associated with the 75° corrugated sheet indicates stronger bistability and a higher energy barrier for transitioning between stable states. In contrast, the 45° corrugated sheet has a shallower energy well, requiring less transition energy and therefore being more susceptible to the loss of its bistable behavior. This energy barrier is potentially important for energy absorption and stiffness tuning design based on this corrugated structure.

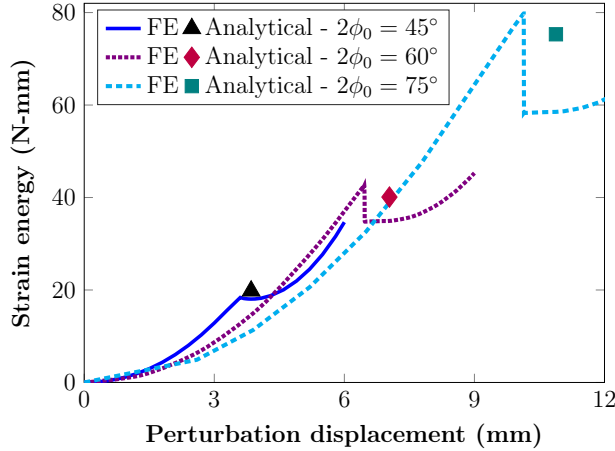


Figure 11: Strain energy of the scar shape for different corrugation angles.

5.2. Aspect ratio

In the previous section, the effect of the corrugation angle on the curvature distribution on the entire geometry was investigated for a fixed aspect ratio of 5. For all corrugation angles, the FE results show that the normalized transverse curvature tends to exceed unity near the boundary for a given aspect ratio. Therefore, this section investigates the influence of aspect ratio on the transverse curvature. A detailed investigation on the variation of longitudinal curvature with aspect ratio is presented in Appendix A; the dependence of bistability on aspect ratio is also explored in the next section.

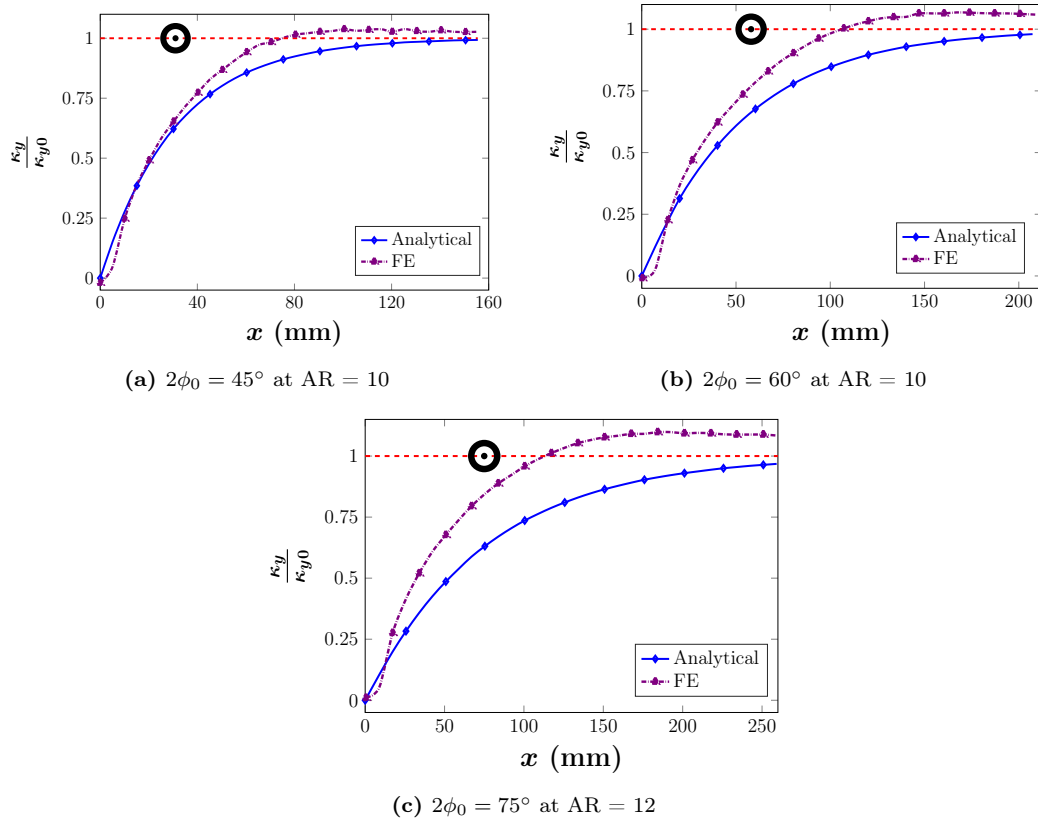


Figure 12: Transverse curvature along the centerline for different corrugation angles and aspect ratios: black circles are theoretical ploy length, $L_p = b/(\pi \lambda)$.

Figure 12 illustrates the normalized transverse curvature variation for all considered geometries at a higher aspect ratio, along with the theoretical

ploy length indicated by a black circle (Seffen et al., 2019; Boddapati et al., 2024). A relatively steeper curvature growth along the unit strip center-line is observed for a smaller corrugation angle, i.e, at $2\phi_0 = 45^\circ$, and the growth in transverse curvature is slower with an increase in corrugation angle. Also, the normalized transverse curvature converges to a value slightly above unity. It is expected that Region *II* exerts a compressive reaction force on Region *I*, leading to higher curvatures than in the original configuration. This expectation is supported by FE simulations, which predict converged normalized curvatures approximately 3%, 6%, and 9% higher than the initial curvature for $2\phi_0 = 45^\circ$, 60° , and 75° , respectively. The analytical model does not capture this increase because of zero ϵ_y assumptions over the entire geometry.

6. Bistable region

It is observed in the previous section that the scar bistability in the corrugated sheet is material independent. Furthermore, the final curvature depends on the aspect ratio, indicating that geometric configurations play a significant role in governing the bistable behavior. Therefore, this section investigates the influence of various geometric parameters on bistability and the resulting transverse curvature for an isotropic circular arcs corrugated unit strip, thereby providing design guidelines for developing bistable systems based on corrugated sheets.

6.1. Influence of aspect ratio

The aspect ratio is a key geometric parameter that governs the bistable behavior of a wide range of systems. For instance, Boddapati et al. (2024) demonstrated that clamped prestressed laminates require a critical length to exhibit bistability. Below this critical length, the laminates are unable to recover their initial curvature and, consequently, cannot retain their original stable shape. Therefore, the threshold aspect ratio required to achieve bistability is investigated using the FE model for corrugation angles up to $2\phi_0 = 90^\circ$ for different radius-to-thickness ratios, R/t , as shown in Figure 13. The contour map was generated using Python scripting, with black dots representing the FE data points and the contours constructed through linear interpolation of these data. The automation is based on the strain energy difference between the scar shape and the corrugated sheet. The results indicate that at smaller corrugation angles, bistability can be achieved only

with relatively large R/t ratios. As the corrugation angle increases, the critical R/t required for bistability decreases. Although a minimum R/t ratio is necessary to induce bistability for each corrugation angle, the corresponding threshold aspect ratio on the monostable/bistable boundary remains nearly constant at approximately 3.75 across the investigated parameter range. It is important to note that the search for bistability is limited to aspect ratios up to 15.

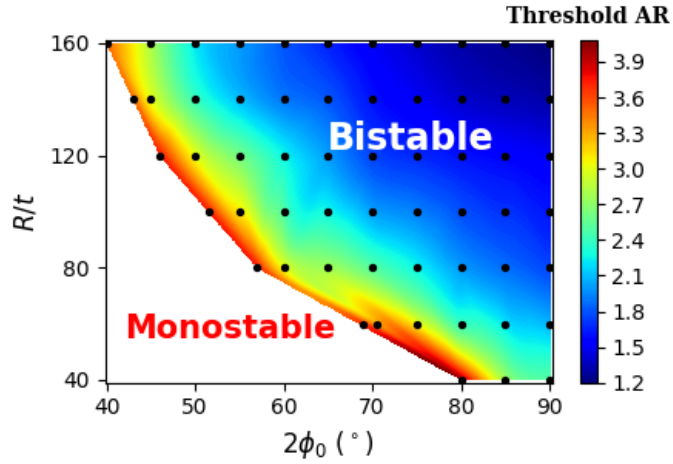


Figure 13: Contour map of threshold aspect ratio for scar bistability in corrugated sheets.

Figure 14(a) depicts the critical aspect ratio for bistability in corrugated sheets for a particular radius-to-thickness ratio of 80. The FE analysis shows that the minimum corrugation angle required to induce bistability, irrespective of the aspect ratio, is 57° , for which the corresponding critical aspect ratio is 3.75. As the corrugation angle increases, the critical aspect ratio decreases; for instance, bistability can be achieved at an aspect ratio of 1.75 for $2\phi_0 = 90^\circ$. Once bistability is achieved, the transverse curvature at the free end is predicted at the critical AR using both the analytical and FE models, as illustrated in Figure 14(b). The analytical prediction of transverse curvature closely follows the FE results. For a corrugation angle of 57° , the recovered transverse curvature at the free edge is approximately 96% of its initial value according to the FE analysis and 94% according to the analytical model. Furthermore, the recovered normalized transverse curvature at the critical aspect ratio decreases significantly with increasing corrugation angle,

reducing to approximately 40% in the FE analysis and 51% in the analytical prediction for $2\phi_0 = 90^\circ$. Unlike prestressed clamped bistable laminates (Boddapati et al., 2024), corrugated sheets do not require full recovery of the initial transverse curvature at the free edge to achieve bistability.

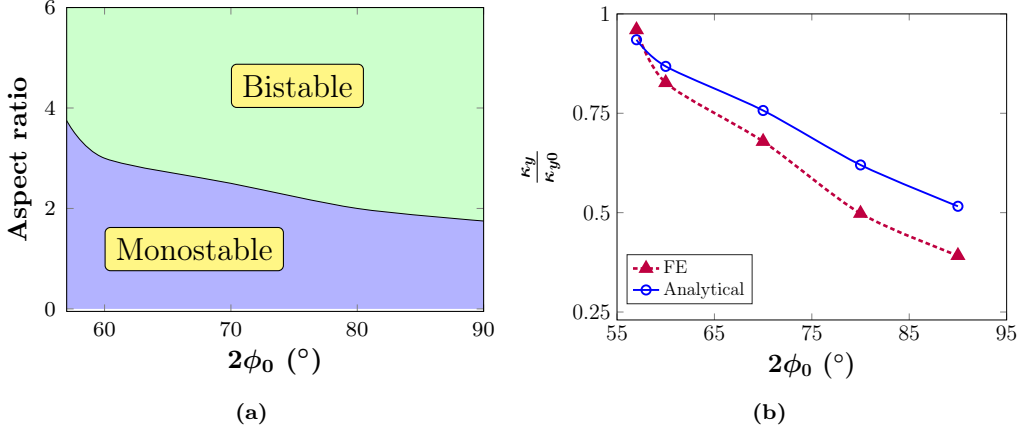


Figure 14: For a fixed $R/t = 80$: (a) stability phase diagram, and (b) normalized free-end transverse curvature prediction for varying corrugation angles at the critical AR.

6.2. Influence of asymmetric geometry

The corrugation unit strip has 2 regions with opposite curvatures, each with radius R_1 or R_2 and subtending angles $2\phi_1$ or $2\phi_2$, respectively. In previous sections, the two regions were assumed to have identical radii and subtended angles. This section explores the bistable behavior of the asymmetric unit strip. The analyses are performed for a fixed aspect ratio of 5 and a shell thickness of 0.125mm. A conventional tape spring, which can be viewed as a corrugated unit strip without Region *II*, does not exhibit scar bistability. Thus, the presence of Region *II* is essential for stabilizing the scar formed in Region *I*. Therefore, investigating the contribution of Region *II* is crucial for understanding the bistability of the corrugated shell.

6.2.1. Asymmetric corrugation angle

Figure 15(a) presents the stability map of the asymmetric corrugated unit strip when varying the subtended angle of Region *I* and Region *II*, while maintaining the same radius of curvature. For a given radius-to-thickness ratio, i.e. 160, the critical corrugation angle required for bistability in the

symmetric configuration is $2\phi_1 = 40^\circ$ (Figure 13). At this critical angle, $2\phi_2$ must be at least 40° to achieve bistability in the unit strip. With an increase in $2\phi_1$, the minimum required corrugation angle in Region *II* decreases; for instance, bistability can be achieved for $2\phi_2$ as low as 12° in a unit strip with $2\phi_1 = 90^\circ$. Below this, the unit strip exhibits monostability. Figure 15(b) presents the predicted transverse curvature at the free end along the centerline when $2\phi_2$ is at its lowest critical value for any given $2\phi_1$. For $AR = 5$, the FE results predict that the normalized transverse curvature at the free end reaches unity for all values of $2\phi_1$. However, the analytical model prediction is lower than the FE prediction, and the discrepancy between the two models increases with increasing $2\phi_1$.

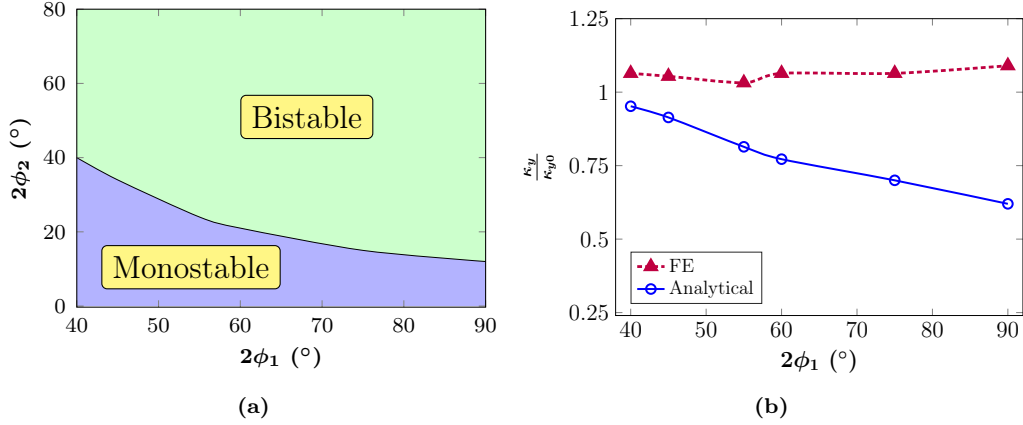


Figure 15: For a fixed $AR = 5$: (a) stability phase diagram, and (b) normalized curvature prediction when $2\phi_2$ is at its lowest critical value for any given $2\phi_1$.

6.2.2. Asymmetric corrugation radius

A parametric study is carried out to investigate the stability map and free-end transverse curvature of an asymmetric unit strip, when the radius of curvature varies while the subtended angle in both regions is kept constant. Figure 16(a) depicts the corresponding stability map for a fixed corrugation angle of 60° and a radius-to-thickness ratio of 160. It is observed that for a fixed corrugation angle, the critical Region *II* radius required for bistability remains nearly constant for a varying Region *I* radius. This observation highlights the dominant role of the corrugation angle in governing bistability, indicating that bistability is primarily controlled by the amplitude-to-span

ratio (which depends only on $2\phi_0$) rather than the radius of curvature. Further, the free-end transverse curvature is predicted using both the FE and analytical models for radius combinations on the monostable/bistable boundary at $AR = 5$, as shown in Figure 16(b). As in the previous section, the FE results indicate that the normalized free-end transverse curvature along the centerline is close to unity for all critical radius combinations. The analytical model exhibits the same overall trend as the FE predictions, with a more pronounced discrepancy at higher corrugation radii.

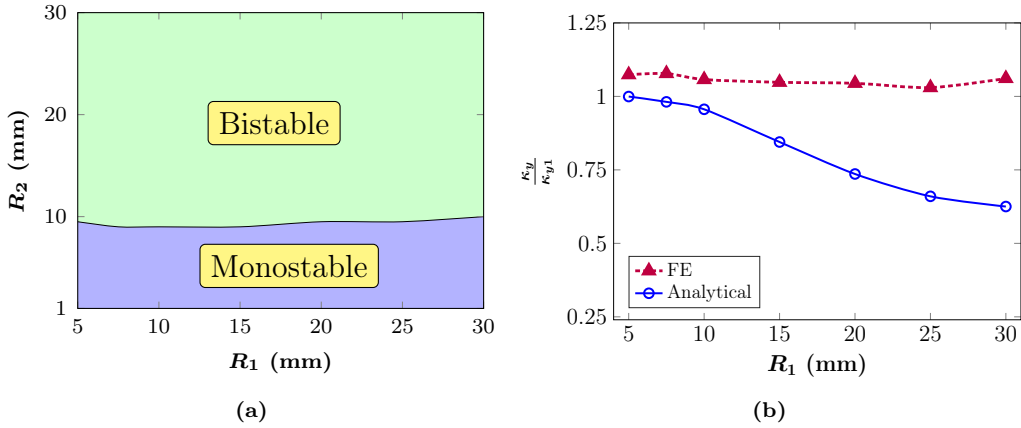


Figure 16: For a fixed $AR = 5$: (a) stability phase diagram, and (b) normalized curvature prediction at critical corrugation radius combinations.

7. Stability of multi-cell structures with cell interaction

In this section, the analytical model is extended to predict the curvature of the scar shape in multi-cell corrugated sheets. In the analytical model, a single unit strip is divided into four segments, two of which belong to Region *I* with positive curvature and the remaining two to Region *II* with negative curvature. Similarly, a multi-cell corrugated structure consisting of N unit strips is divided into $4N$ segments: $2N$ in Region *I* and the remaining $2N$ in Region *II*. The assumption that localization occurs in one corrugation at the expense of the others is based on the work of Meeussen and Van Hecke (2023). The scar is confined to a single corrugation; additional point loads can independently form scars in adjacent corrugations. Adjacent scars can interact after formation, but that interaction is omitted for simplicity.

Figure 17(a) shows a corrugated sheet consisting of three unit strips, where cell P is subjected to a point load to obtain the scar shape shown in Figure 17(b). As in the unit strip analysis, it is assumed that only the two segments adjacent to the point of application of the load experience a significant change in curvature. In contrast, the remaining $4N - 2$ segments undergo an inappreciable change in curvature. The curvature change in these two segments results in a non-uniform axial strain component, ϵ_x , whereas ϵ_x remains constant along y in the remaining $4N - 2$ segments. Consequently, the expression for the total strain energy is modified as:

$$\begin{aligned}
U_{b_P} &= \frac{D}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} [\kappa_x^2 + 2\kappa_x(\kappa_y - \kappa_{y0}) + (\kappa_y - \kappa_{y0})^2] dy dx, \quad U_{b_{\text{rest}}} = 0 \\
U_{s_P} &= \frac{A}{2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{b}{4}}^{\frac{b}{4}} \epsilon_{x_P}^2 dy dx, \quad U_{s_{\text{rest}}} = (4N - 2) \frac{A}{2} \frac{b}{4} \int_{-\frac{L}{2}}^{\frac{L}{2}} \epsilon_{x_{\text{rest}}}^2 dx \\
U(\lambda) &= U_{b_P}(\lambda) + U_{s_P}(\lambda) + U_{s_{\text{rest}}}(\lambda)
\end{aligned} \tag{27}$$

where ϵ_{x_P} and $\epsilon_{x_{\text{rest}}}$ refer to ϵ_{x_I} and $\epsilon_{x_{II}}$, respectively, as defined for the unit strip in Eq.21.

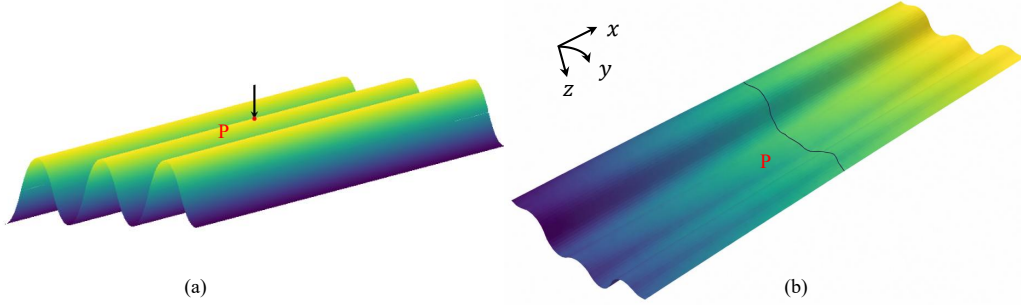


Figure 17: Scar bistability in a multi-cell corrugated sheet: (a) initial corrugated configuration; (b) corresponding stable scar configuration.

Based on the analytical formulation developed for multi-cell corrugated sheets, the transverse curvature distribution along the centerline is predicted and compared with FE results for $N = 3$ and $N = 5$ unit strips. The multi-cell sheets have an opening angle of $2\phi_0 = 60^\circ$, a radius of $R = 20\text{mm}$, and an aspect ratio of 5. Figure 18 compares the analytical predictions with the corresponding FE results. The analytical model agrees well with the FE predictions over most of the centerline. However, as boundary effects become

significant near the free edges, the analytical solution deviates from the FE results because these effects are not accounted for in the present formulation. Furthermore, the FE results indicate that the normalized transverse curvature at the free end is significantly greater than unity and increases with the number of unit strips. For instance, increases of approximately 21% and 25% are observed for structures with $N = 3$ and $N = 5$ unit strips, respectively. The analytical model does not effectively capture this multi-cell interaction as it assumes zero transverse membrane strain ($\epsilon_y = 0$) throughout the geometry.

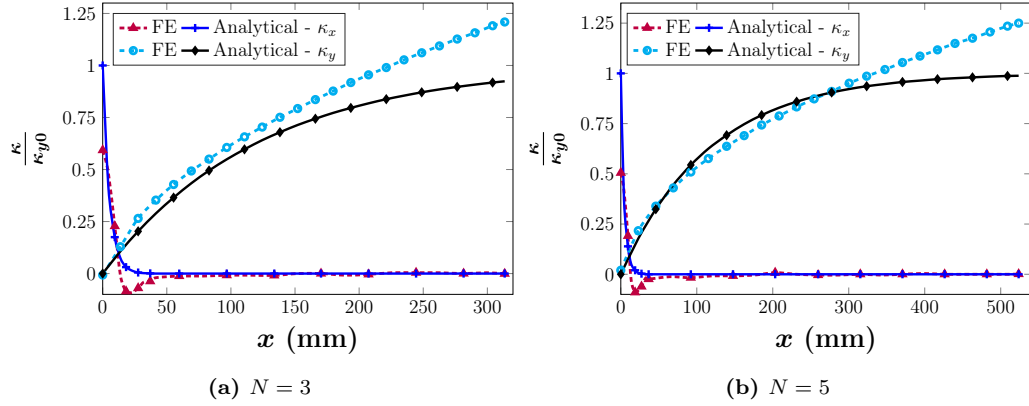


Figure 18: Curvatures along the centerline scar shape of a multi-cell corrugated sheet with $R = 20\text{mm}$ and $2\phi_0 = 60^\circ$ for a varying number of unit strips.

Furthermore, the normalized longitudinal curvature at the center is approximately 0.6 and 0.5 for the multi-cell with $N = 3$ and $N = 5$, respectively. These values are considerably lower than the value of 1 predicted by the local simplified model. This behavior is expected since the local simplified model prediction (Eq.6) was derived by idealizing the corrugated unit strip as a tape-spring problem and is therefore not directly applicable to a multi-cell corrugated sheet. In multicellular structures, interactions between adjacent cells reduce longitudinal curvature, resulting in values that are lower than predicted. Furthermore, as the number of unit strips increases, the discrepancy between the analytical prediction based on the local simplified model and the FE results becomes increasingly pronounced. The FE results predict that the longitudinal curvature is reduced by a factor of α relative to the simplified model, where $\alpha < 1$ and depends on the number of unit strips. Accordingly, Eq.17 need to be modified to incorporate this reduction, and

the remaining governing equations can be updated. Developing a predictive expression for α as a function of the number of unit strips remains a potential direction for future research.

8. Conclusion

This work presented an analytical framework for investigating scar bistability in circular arcs corrugated sheets. The model predicted the curvature evolution across the entire geometry as the sheet transitions from a globally flat corrugated sheet to a localized scar configuration. Scar bistability in corrugated sheets was first demonstrated experimentally, and the FE model was validated by comparing the predicted and measured scar shapes in composite corrugated sheets. The validated FE model was then used to compare the results of the proposed analytical model over a wide range of geometric and material parameters. The robust analytical model depends on only one unknown parameter, determined by minimizing the total strain energy, including stretching, bending, and bending–stretching coupling. The model accurately captures the decay of longitudinal curvature and the growth of transverse curvature. Several conclusions arise from the analytical parametric study. In composite corrugated sheets, the curvature of the scar shape is influenced by both material and geometric properties. However, in an isotropic material, the normalized longitudinal curvature decay is independent of material and geometric properties except the aspect ratio. In contrast, transverse curvature growth depends on the corrugation geometry. Furthermore, the converged normalized transverse curvature at the free ends is slightly greater than the initial curvature, and this increase becomes more pronounced as the corrugation angle increases.

Scar bistability requires a minimum aspect ratio, and this threshold depends on both the corrugation angle and the radius-to-thickness ratio. The models predict that, unlike previously reported bistable systems, the transverse curvature at the free end need not fully recover its initial value for the corrugated unit strip to attain bistability, which has important design implications for shape-morphing scenarios utilizing multiple scars per corrugation. Parametric studies were also conducted to investigate the role of the curved side regions (Region *II* in the paper) surrounding the central tape-spring-like structure on bistability. The results revealed that bistability requires a minimum corrugation angle and radius in these side regions. The critical corrugation angle depends on the corrugation angle in the central

tape-spring-like region (Region *I*), whereas the critical corrugation radius is nearly independent of the corrugation radius in Region *I*. For these critical geometric conditions, the proposed analytical model predicted the free-end curvature for a fixed aspect ratio, and captured the trend obtained with the FE simulations.

The analytical framework was further extended to predict bistability in multi-cell corrugated sheets with interactions between adjacent cells. The model assumed that the change in curvature was primarily localized within the cell directly beneath the applied point load, resulting in a spatially varying longitudinal membrane strain in both in-plane directions. In the remaining cells, the curvature change was assumed to be negligible, leading to an approximately constant longitudinal membrane strain in the transverse direction. The FE results indicated that the longitudinal curvature using the local simplified model was no longer applicable to multi-cell structures due to cell interactions. Nevertheless, the global analytical model predicted the transverse curvature with good accuracy, with noticeable discrepancies appearing only near the free end, where boundary effects are also dominant. The FE simulations also showed that cell interactions significantly increase the normalized transverse curvature at the free end. This increase was not fully captured by the analytical model, highlighting the need to incorporate multi-cell interaction effects into the theoretical formulation. Future work will focus on developing a more rigorous analytical formulation of longitudinal curvature and on incorporating deep shell theory into the derivation of transverse curvature. In addition, incorporating cell-cell interaction in the analytical framework is a potential research opportunity. Finally, the ultimate goal is the use of this model in a larger framework for the design and discovery of new morphing structures utilizing scars to derive new functions.

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Appendix A. Decay parameter ξ for the longitudinal curvature

This section presents a detailed parametric study conducted using the FE model to investigate how decay parameter for longitudinal curvature ξ varies with these geometric parameters for an isotropic material. Figure A.19(a) depicts the variation of ξ as a function of the aspect ratio while keeping all other geometric parameters constant. The decay parameter ξ linearly increases with the aspect ratio, indicating that the curvature decays more rapidly at higher aspect ratios. Similarly, the sensitivity of ξ to the remaining geometric parameters is investigated and shown in Figure A.19(b). The results demonstrate that ξ remains largely insensitive to other geometric parameters. Thus, the decay parameter can be determined solely from the aspect ratio and treated as a constant for any parametric study in which the aspect ratio remains unchanged.

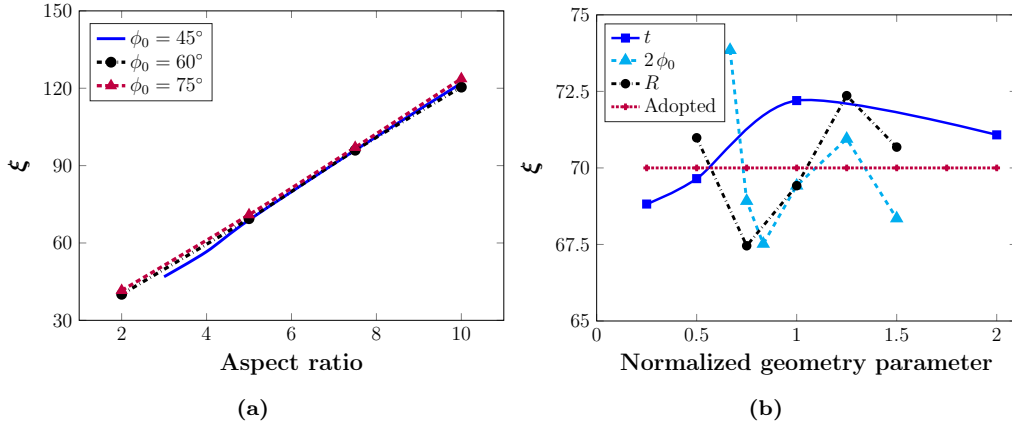


Figure A.19: Variation of decay parameter ξ with (a) aspect ratio and (b) other geometric parameters.

Figure A.20(a) and (b) present the approximated longitudinal curvature along the x -direction at $y = 0$, used as the boundary condition in Eq.16, obtained for various aspect ratios and corrugation angles. As illustrated in Figure A.19(a), the longitudinal curvature is highly sensitive to the aspect ratio; it decays rapidly with an increase in the aspect ratio. Conversely, Figure A.20(b) reveals that the decay rate is largely insensitive to variations in the initial corrugation angle.

For composite corrugated sheets, the initial longitudinal curvature depends on the material properties of individual laminae through the bending

stiffness tensor $\mathbf{D}^*$. To investigate the dependence on decay nature along the length, a parametric study is conducted on the decay parameter ξ across normalized lamina material properties for a $[90/0/90]$ composite corrugated sheet, as shown in Figure A.21. Normalized material parameters are varied from 0.5 to 1.5 while maintaining the remaining material properties listed in Table 1. The results indicate that ξ remains nearly constant across variations in material properties. Consequently, the longitudinal curvature decay behavior of composite corrugated sheets remains invariant, justifying the treatment of ξ as a constant even in composite corrugated sheets.

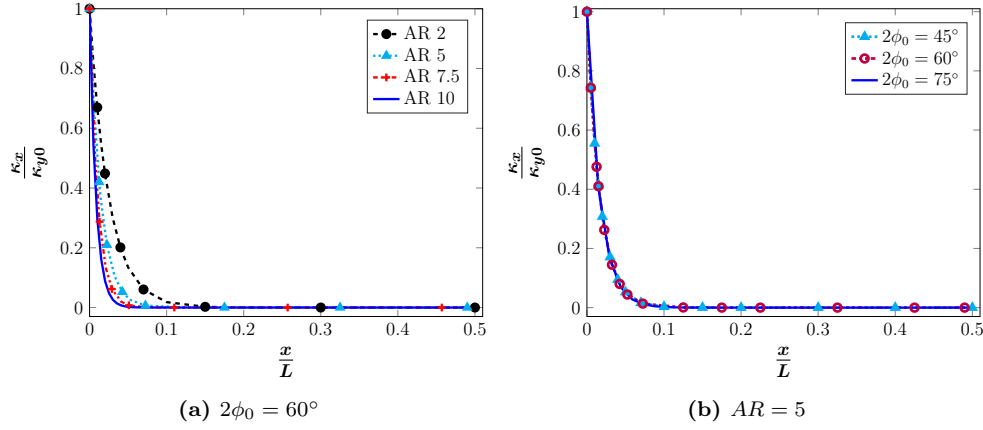


Figure A.20: Variation of approximated longitudinal curvature along x at $y = 0$ with (a) aspect ratio and (b) corrugation angle.

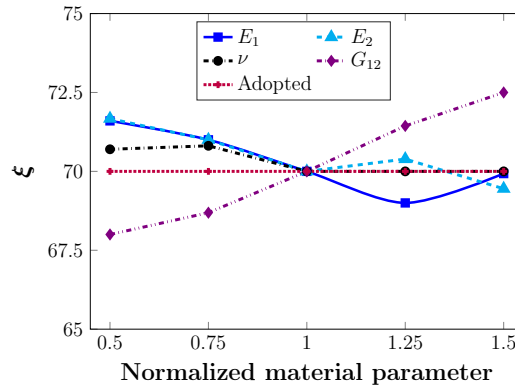


Figure A.21: Variation of decay parameter ξ with normalized material properties of composite lamina, for $2\phi_0 = 60^\circ$, $R = 20\text{mm}$, $t = 0.125\text{mm}$ and $AR = 5$.

Appendix B. Longitudinal curvature using shallow shell theory

From Donnell's shell theory (Amabili, 2005), valid for a shallow shell, the curvature changes are related to the displacements by:

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2}, \quad \kappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y} \quad (\text{B.1})$$

Following the tape-spring analogy similar to Seffen et al. (2019), the twisting and longitudinal curvatures can be derived from the Codazzi–Mainardi compatibility equations by assuming zero twisting curvature and an exponentially decaying longitudinal curvature, respectively, yielding:

$$\begin{aligned} \kappa_{xy} &= \lambda \kappa_{y0} \exp\left(-\lambda \frac{\pi x}{b}\right) \sin\left(\frac{\pi y}{b}\right) \\ \kappa_x &= \kappa_{y0} \exp\left(-\xi \frac{x}{L}\right) - \kappa_{y0} \lambda^2 \exp\left(-\lambda \frac{\pi x}{b}\right) \left(1 - \cos\left(\frac{\pi y}{b}\right)\right) \end{aligned} \quad (\text{B.2})$$

Figure B.22 compares the transverse curvature prediction using shallow shell and deep shell theory with the FE analysis for $R = 20\text{mm}$, $2\phi_0 = 45^\circ$, and aspect ratio of 5 for a Be-Cu alloy sheet. The results demonstrate that shallow-shell theory is insufficient to accurately predict the transverse curvature and the resulting scar bistability; therefore, deep-shell theory is necessary to explain the scar bistability in corrugated sheets.

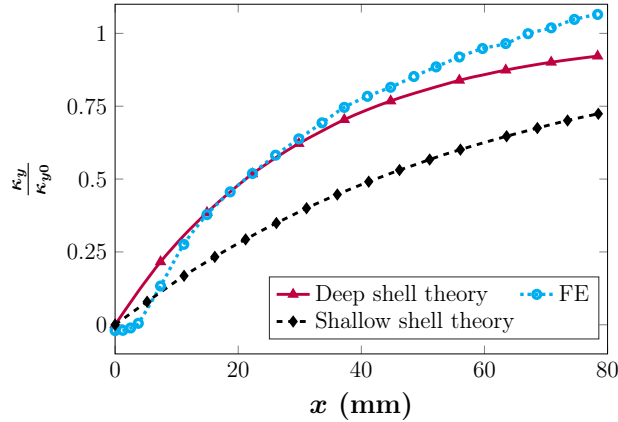


Figure B.22: Transverse curvature along x at $y = 0$ predicted using deep and shallow shell theories.

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